



# *Mathematical Methods*

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## *Outline*

- Introduction
- Problem Statement
- Terminology
- Overview of methods
  - Stone-Byron

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*Introduction*



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*Introduction*

“[I]f we put all these initial estimates together [for a SAM] we reach the familiar situation: an incomplete and inconsistent social accounting matrix”  
(Stone, 1977, p xxiv).

- Infinite number of possible solutions to achieve a consistent SAM
- Even with the best efforts to collect complete data, the prior will be inconsistent and may be incomplete
- Mathematical methods cannot make the prior complete

**BUT mathematical methods may be useful**



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***Problem Statement***



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***Incomplete and Imperfect Information***

*“[I]t is **impossible** to establish by direct estimation a system of national accounts free of statistical discrepancies, residual error, unidentified items, balancing errors and the like since the information available is in some degree **incomplete, inconsistent and unreliable**. Accordingly, the task of measurement is not finished when the initial estimates have been made and remains incomplete until final estimates had been obtained which satisfy the constraints that hold between their true values.”*

*(Stone, 1982, p 186, emphasis added).*



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 **Accuracy and Relevance**

*“The issue of whether the SAM is deterministic or stochastic is crucial as the SAM provides the underlying data set upon which simple SAM-multiplier analyses and more complex Computable General Equilibrium Models (CGEs) are calibrated. Increasingly, these models are used to explore and simulate the impact of policies and exogenous shocks on the whole socio-economic system. An erroneous or inaccurate SAM invalidates the results obtained from these models”  
(Thorbecke, 2003, p 186).*

SAM as Accounts, 2026 © cgemod  7

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 **Objective**

**A complete, consistent and accurate SAM**

<b>Consistent:</b> every expenditure has a matching and equal income	<b>Reconciled Balanced</b>
<b>Complete:</b> all transactions recorded	<b>How can we know?</b>
<b>Accurate:</b> TVs accurately reflect the system	<b>How can we know?</b>

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 **Objective**

**If Complete and Consistent**  
Incomes equal expenditures for ALL agents

**If *Incomplete* and Consistent**  
Incomes equal expenditures for ALL agents  
**BUT some transactions are biased**

**If Complete and *Inconsistent***  
Incomes NOT equal expenditures for ALL agents

**If *Inaccurate***  
Policy experiments will be ‘misleading’



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 **Problem**

**Estimating a complete, consistent and accurate SAM**

**From**  
Incomplete data  
Inconsistent data  
Unreliable data

**Wanted:**  
A theoretically grounded method that uses all the available information, e.g., point estimates, error bounds, SEs, etc., without making ‘strong’ assumptions, e.g., normal error distributions.

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*Terminology*



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*Terminology*

- **Balanced**
  - A consistent matrix derived from prior (current) data
- **Reconciled**
  - A consistent matrix derived from prior (current) data and other constraints
- **Updated**
  - A consistent matrix derived from a past matrix
- **Estimated**
  - An estimated matrix that is consistent with prior (current) data (point estimates and error bounds)

**Balancing and Reconciling methods may be reasonable if the inconsistencies are ‘small’**

**Estimation methods needed if the inconsistencies are NOT ‘small’**

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*Overview of methods*



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*Some Methods*

- RAS
  - Modified RAS
  - RAS ‘with errors’
- Squared differences
  - Weighted
  - Unweighted
- Stone-Byron
- Loss functions
- Entropy

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## *Stone-Byron*

$$F = \sum_{ij} \frac{(x_{ij}^* - x_{ij})^2}{v_{ij}}$$

$v_{ij}$  = variances  
 $x_{ij}$  = prior and solution (\*) TVs

**Requires: estimates (subjective) of the variances**

“The solution so obtained would give the minimum possible value of the sum of the squares of the standardised errors. Also, in cases where the error distributions are normal this solution would satisfy the criterion of maximum likelihood. In general, however, it will not be possible in what follows to measure the standard errors in the ordinary way and we shall have to resort to setting margins of error, which will be denoted by  $\zeta_j$ , such that they are the best estimates we can make of parameters analogous to standard errors”  
(Stone, *et al.*, 1942, p 112).

**Normally distributed errors → ML**

$$L = \left[ \frac{T}{\sqrt{(2\pi)}} v_{ij} \right] \exp \left[ -\frac{(x_{ij}^* - x_{ij})^2}{2v_{ij}} \right]$$

$T$  = number of elements 

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# *Mathematical Methods*

The End



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