Price Changes and Consumer Welfare

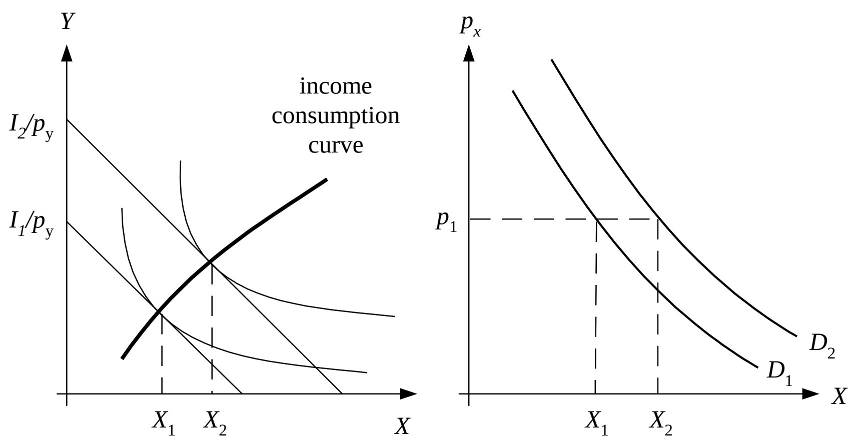
While the basic theory previously considered is extremely useful as a tool for analysis, it is also somewhat restrictive. The theory of consumer choice is often referred to as 'going behind the demand curve', but in elementary price theory extensive use is made of the *ceteris paribus* assumption, e.g.,

$$X = f\left(p_x\right)_{\bar{I},\bar{p}_y} \tag{1}$$

where $\bar{I}$ indicates constant money income, a $\bar{p}_y$ indicates constant price for *Y*.

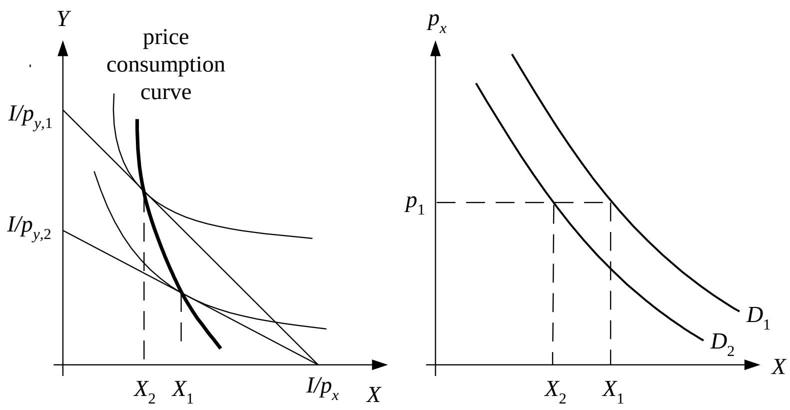
At first sight, this presents no great difficulties for our analysis if money income changes. If money income increases and both *X* and *Y* are normal goods then the demand curve for *X* will shift to the right, although if *X* is an inferior good its demand curve will shift to the left.

Figure 1 Income Changes and Demand Curves



However, reality, or precision if preferred, is not so simple. To see this, consider the effect of a change in p_y on the demand for *X*. For example, assuming that p_y declines, *I* is constant and that *X* and *Y* are substitutes then the following is possible.

Figure 2 Prices Changes and Demand Curves

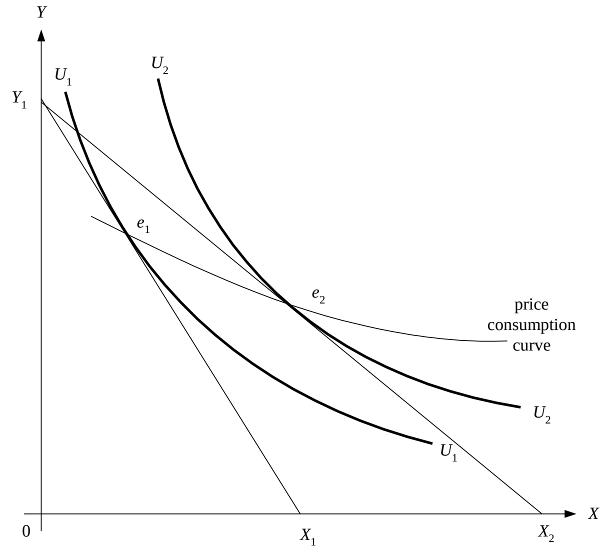


These two cases seem to be telling contradictory stories. First, when money income increased we moved to a higher level of utility, but second, although money remained constant the fall in p_y led to a higher level of utility. Clearly, there is a potential problem with money income as an indicator of the standard of living. To appreciate what is happening we must embrace the concepts of income and substitution effects and the idea(s) of real income.

1. Income and Substitution Effects

Actual consumption decisions in response to changes in income and prices clearly depend upon the specific 'shape' of a household's indifference map, and the consequent price and income consumption curves. This suggests a degree of interdependence between the effects of price and income changes.

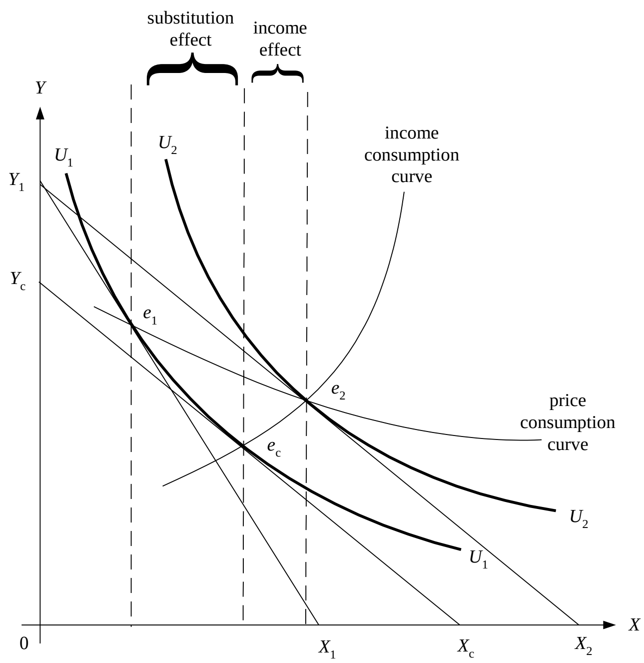
Figure 1.1 Income and Substitution Effects 1



When p_X falls, we move along the price consumption line from e_1 to e_2. This movement can however be decomposed into a substitution effect and an income effect.

Assume we wish to hold utility constant at the initial level, U_1, but at the new, lower, price for X. Given our assumptions about the shape of indifference curves we can identify a 'budget constraint' that is tangential to U_1 but with the slope of the new budget constraint Y_1X_2, i.e., that existing after the change in p_X, that we will label Y_cX_c.

Figure 1.2 Income and Substitution Effects Final



The movement from e_1 to e_c is known as the *substitution effect*, consequent upon the price change when holding utility constant, while the movement from e_c to e_2 is known as the *income effect* consequent upon a real income change when holding prices constant.

Note that the substitution effect will always be positive for a fall in price and negative for a rise in price. But the sign on the income effect will depend upon whether the good is normal or not. For a normal good the income effect will be positive, and for an inferior good it will be negative. It is this combination of income and substitution effects that gives rises to possibility, however remote, of a Giffen good. A simple table can be used to classify goods by the 'total effects' of a price increase for a good in terms of the 'substitution effects' and 'income effects', i.e.,

Impact of an Increase in the Price of a Good

Substitution Effect	Income Effect	Total Effect	Type of Good
Quantity decreases	Quantity decreases	Quantity decreases	NORMAL
Quantity decreases	Quantity increases	Quantity decreases	INFERIOR
Quantity decreases	Quantity increases	Quantity increases	GIFFIN

2. Slutsky Equation

The Slutsky equation, named after Eugene Slutsky the Russian economist who first defined the expression before the October Revolution, provides a formal statement of the total, income and substitution effects,

Total Effect = Substitution Effect + Income Effect

Or more formally

$$\frac{\Delta X}{\Delta p_x} = \left(\frac{\Delta X}{\Delta p_x}\right)_{comp} + \textit{Income effect} \qquad (2)$$

where ΔX is the quantity change associated with a given price change, Δp_x, and 'comp' is the 'compensated response', or substitution effects associated with the price change. How can the 'income effects' be represented? The income effect consists of two elements: the change in the quantity demanded per unit of 'extra' income and the amount of the extra income, i.e.,

$$\frac{\Delta X}{\Delta p_x} = \left(\frac{\Delta X}{\Delta p_x}\right)_{comp} + \left(-X_1 . \frac{\Delta X}{\Delta I}\right). \qquad (3)$$

The response component is simply $\Delta X/\Delta I$, whereas X_1 is the income component, where X_1 is the amount of the good that can be purchased with a unit of currency, and is negative to indicate that a price increase reduces income to be spent elsewhere.

What matters here are the signs on the various components of the Slutsky equation, because of what they tell us about the income and substitution effects. The substitution effect, $(\Delta X/\Delta p_x)_{comp}$, is always negative, so the real interest lies in the income effect.

i) If the good is a NORMAL good then $(\Delta X/\Delta I) > 0$, by definition, and the net impact of the income effect is negative. Therefore for a NORMAL good the good necessarily obeys the *Law of Demand*.

ii) If the good is an INFERIOR good then $(\Delta X/\Delta I) < 0$, by definition, and the net impact of the income effect is positive. Therefore for impact of a price increase for an INFERIOR good depends upon the relative magnitudes of the substitution effect and the income effect.

   a) If the negative substitution effect is greater than the positive income effect we have an INFERIOR good that conforms to the *Law of Demand*.

   b) If the negative substitution effect is less than the positive income effect we have an INFERIOR good that does not conform to the *Law of Demand*, i.e., a GIFFIN good.

Notice that the magnitude of the income effect depends crucially upon the magnitude of X_1 relative to total income. Hence the income effect of a price increase will be directly related to the importance of the good in a consumer's consumption bundle.

3. Compensating and Equivalent Variations

It should be evident that changes in prices can induce changes in well-being. We might wish to measures these in terms of *utils,* but such measures have no meaning because they are subjective, and therefore cannot facilitate inter personal comparisons. On the other hand, the 'income effect' appears to provide a theoretical means by which we can give substance to the change in well-being in terms of units of a currency.

There are two important methods for addressing this issue.

Compensating Variation

The monetary cost of restoring an individual to an initial level of utility ***after*** an increase in the price of a good, i.e., compensating variation in money income to allow for price change.

Equivalent Variation

The income reduction that is equivalent in its effect on welfare to an increase in the price of a good, i.e., the variation in money income equivalent to the price change.

These are initially somewhat confusing. It is important to bear in mind that the emphasis is on quantifying the changes in welfare that arise from increases/decreases in prices, e.g., we will use these ideas to underpin the economic interpretation of index numbers; these ideas are also using in quantitative policy analyses; etc.

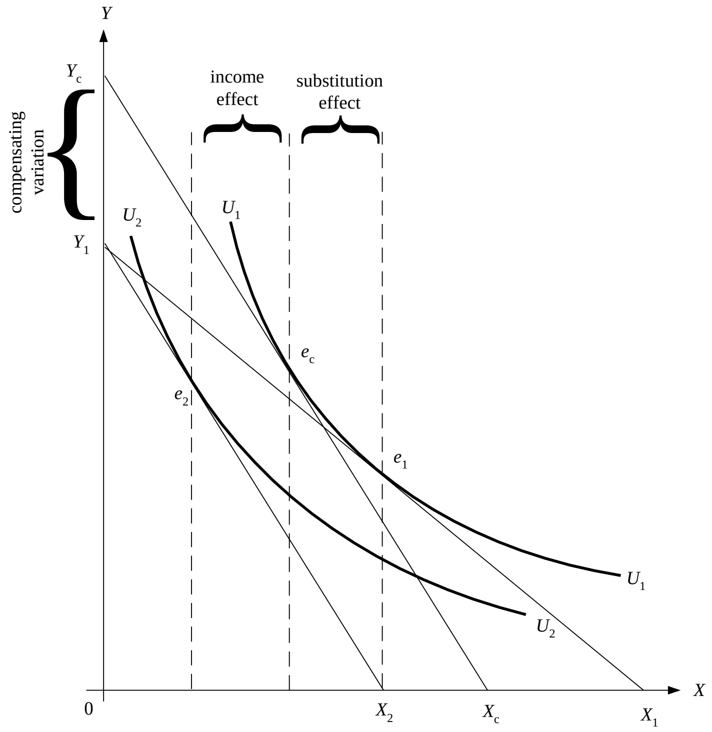
3.1 Hicks Compensating and Equivalent Variations

Defining Real Income in terms of a constant level of utility is a concept attributable to Hicks, and hence is known as Hicks real income. The income and substitution effects and compensating and equivalent variations analysed using the Hicks and real income definition are therefore termed, respectively, Hicks income and substitution effects and Hicks compensating and equivalent variations.

Hicks Compensating Variation

For the compensating variation, the benchmarks for our analyses are the ***initial*** levels of utility and the ***new*** prices. The Hicksian approach is based on the representation of income and substitution effects developed above, i.e.,

Figure 3.1 Hicks Compensating Variation



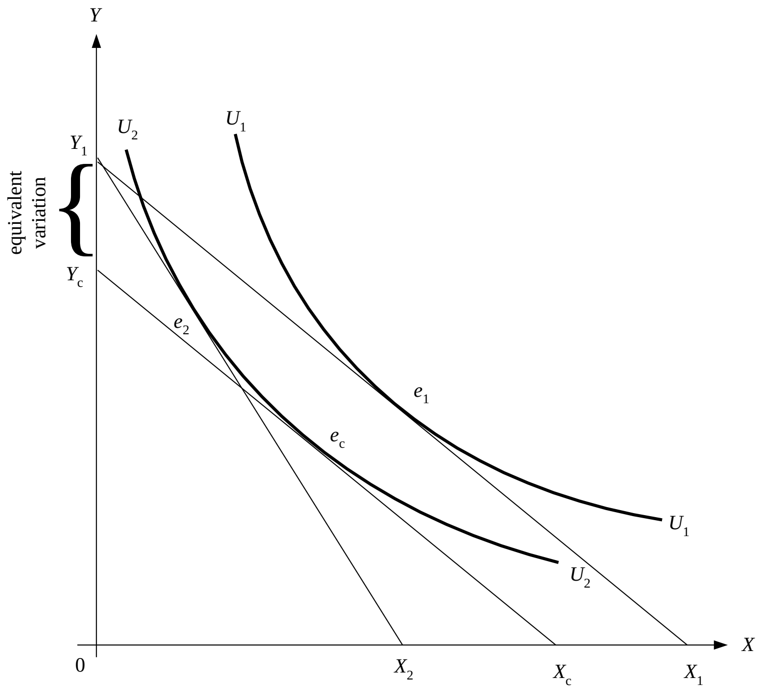
Starting at e_1 with indifference curve U_1 and budget constraint Y_1X_1, the consumer faces an increase in p_x, which pivots the budget constraint to Y_1X_2 leading to a new optimum at e_2 on U_2. The total effect can be broken down into the substitution and income effects. Add a hypothetical budget constraint with the same relative prices (slope) as Y_1X_2 but that has a point of tangency with U_1, i.e., Y_cX_c. The transition from e_1 to e_c then represents the effect due to the change in relative prices and the transition from e_c to e_2 represents the income effect.

To restore the consumer to the same level of well-being experienced before the price increase, i.e., to compensate the consumer for the price increase, but at the new prices would require a monetary transfer of (Y_c – Y_1).

Hicks Equivalent Variation

For the equivalent variation the benchmarks for our analyses are the ***new*** levels of utility and the ***original*** prices.

Figure 3.2 Hicks Equivalent Variation



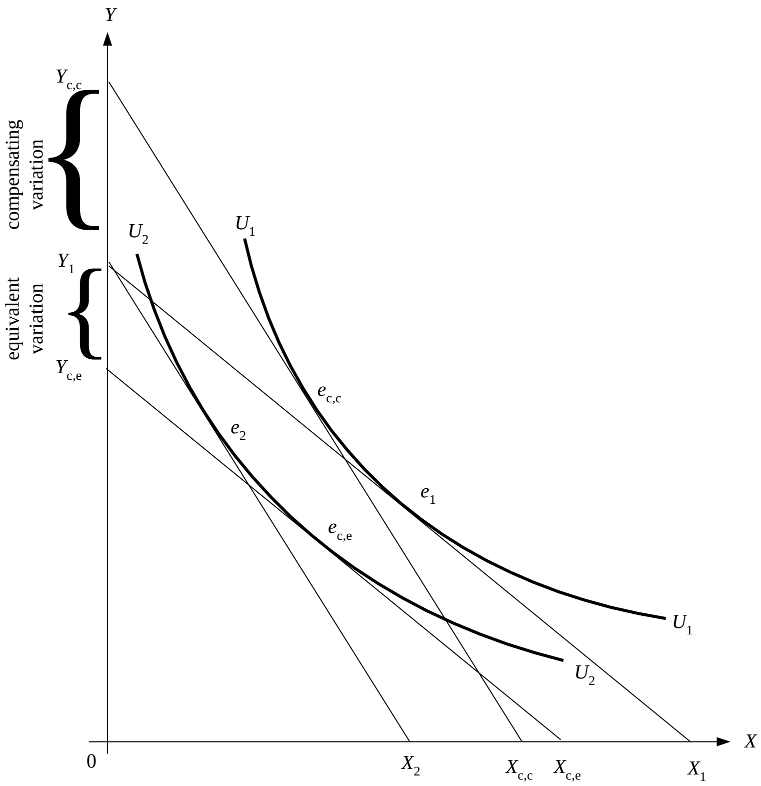
Starting at e_1 with indifference curve U_1 and budget constraint Y_1X_1, the consumer faces an increase in the p_x, which pivots the budget constraint to Y_1X_2 leading to a new optimum at e_2 on U_2. The total effect can be broken down into the substitution and income effects. Add a hypothetical budget constraint with the same relative price (slope) as Y_1X_1 but that has a point of tangency with U_2, i.e., Y_cX_c. The transition from e_2 to e_c then represents the effect due to the change in relative prices, and the transition from e_c to e_1 represents the income effect.

The reduction in income to the consumer equivalent to the reduction in utility associated with the price increase, but at the old/initial prices would require a monetary transfer of (Y_1 – Y_c).

Compensating vv Equivalent Variation

The compensating and equivalent variation estimates of the difference/change in real income are not the same. This is because they are based on different sets of relative prices. The compensating variation measure uses the ***new*** prices while the equivalent variation measure uses the ***original*** prices.

Figure 3.3 Hicksian Compensating and Equivalent Variations Compared

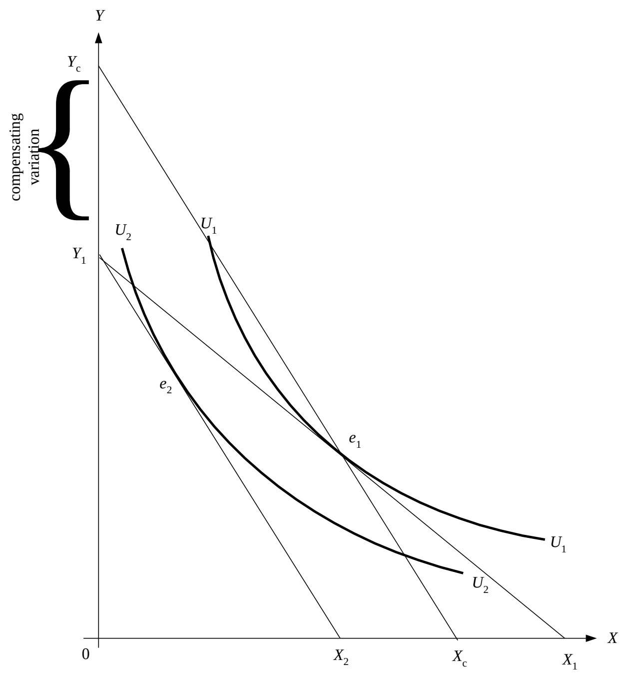


3.2 Slutsky Compensating and Equivalent Variations

There is however a fundamental problem with the Hicks measure of real income: it requires knowledge of the precise shape of the indifference map. This difficulty can be overcome, for practical purposes, by using the approximation known as Slutsky real income. Slutsky real income is the level of money income required to purchase the *initial* bundle of goods at the *new* set of prices. **Constant Slutsky real income is therefore defined by a budget constraint that permits purchase of the initial bundle of goods at the new set of prices**.

We can illustrate the Slutsky compensating variation in a simple diagram.

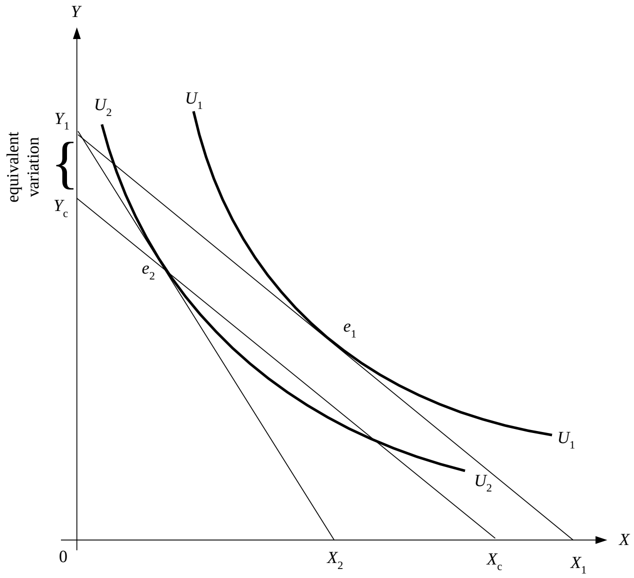
Figure 3.4 Slutsky Compensating Variation



As before p_X increases causing the optimum consumption choice to shift from e_1 to e_2. Constant Slutsky real income is defined by a budget constraint passing through e_1 and parallel to the budget constraint effective at e_2, i.e., with the same slope or relative prices. **The Slutsky compensating variation will always be greater than the Hicks compensating variation for a price increase and less for a price fall**.

Slutsky real income is defined as that level of money income required to purchase the new bundle of goods at the old set of relative prices. Diagramatically

Figure 3.5 Slutsky Equivalent Variation



As with compensating variation versions, the income and substitution effects derived from the two approaches will in general differ. **For a price increase the Slutsky EV will be less than the Hicksian EV, and for a price decrease the Slutsky EV will be greater than the Hicksian EV**.

3.3 Slutsky Equation and Compensating Variation

Consider a simple Slutsky compensating variation diagram. Money income remains the same but the price of *X* declines, causing the budget constraint to pivot around the vertical intercept. We can now calculate the change in income implicit to the change in the price of *X*, by drawing a budget constraint with a slope determined by the new relative prices that has a money income level just sufficient to purchase the original bundle at e_0, i.e.,

$$I_0 = p_{x,0}X_0 + p_{y,0}Y_0$$
$$I_1 = p_{x,1}X_0 + p_{y,0}Y_0$$

and the compensating difference in income is

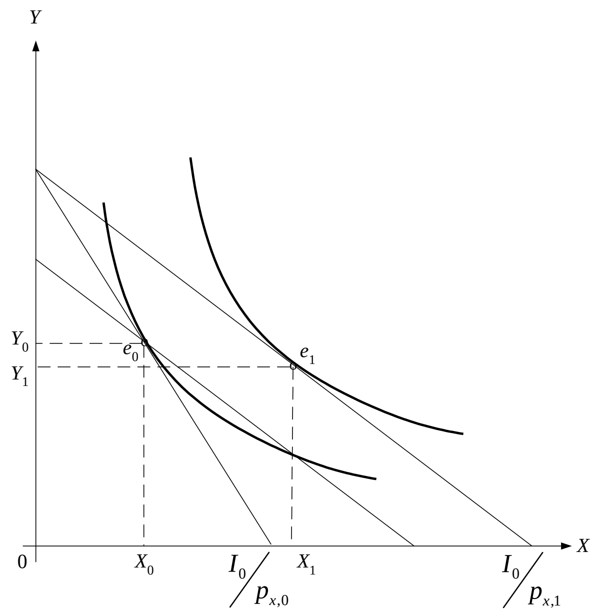
$$\begin{aligned} I_1 - I_0 &= p_{x,1}X_0 + p_{y,0}Y_0 - p_{x,0}X_0 - p_{y,0}Y_0 \\ &= p_{x,1}X_0 - p_{x,0}X_0 \\ &= \left(p_{x,1} - p_{x,0}\right)X_0 \end{aligned}$$

and in discrete change notation

$$\Delta I = \Delta p_x . X_0$$

and hence that the change in price and the change in income both move in the same direction.

Figure 3.6 Slutsky Equation and Compensating Variation



The substitution effect is defined as the change the quantity demanded holding real income constant. Approximating this using the Slutsky approximation is represented in the diagram below. The substitution effect is defined as the change in demand for X and Y given the new prices and an income just sufficient to purchase the old bundle but at the new prices, i.e.,

$$\Delta X_{comp} = x_0\left(p_{x,1}, I_1\right) - x_0\left(p_{x,0}, I_0\right)$$

where lower case x_i indicates the demand function for X_i conditional upon the bracketed terms. The substitution is always negative given the MRS conditions.

The income effect is defined as the change the quantity demanded holding price of X constant at the new prices, i.e.,

$$\Delta X_{inc} = x_0\left(p_{x,1}, I_0\right) - x_0\left(p_{x,1}, I_1\right).$$

Now the total effect is defined as the substitution effect plus the income effect, i.e.,

$$\Delta X = x_0\left(p_{x,1}, I_0\right) - x_0\left(p_{x,0}, I_0\right)$$

defined as the substitution effect plus the income effect, i.e.,

$$\Delta X = \Delta X_{comp} + \Delta X_{inc}$$

and substituting for the income and substitution effects gives

$$\begin{aligned} \Delta X &= \Delta X_{comp} + \Delta X_{inc} \\ \left[x_0\left(p_{x,1}, I_0\right) - x_0\left(p_{x,0}, I_0\right)\right] &= \left[x_0\left(p_{x,1}, I_1\right) - x_0\left(p_{x,0}, I_0\right)\right] \\ &\quad + \left[x_0\left(p_{x,1}, I_0\right) - x_0\left(p_{x,1}, I_1\right)\right] \end{aligned} .$$

We are however interested in how demand changes in response to changes in prices. Thus, we can write

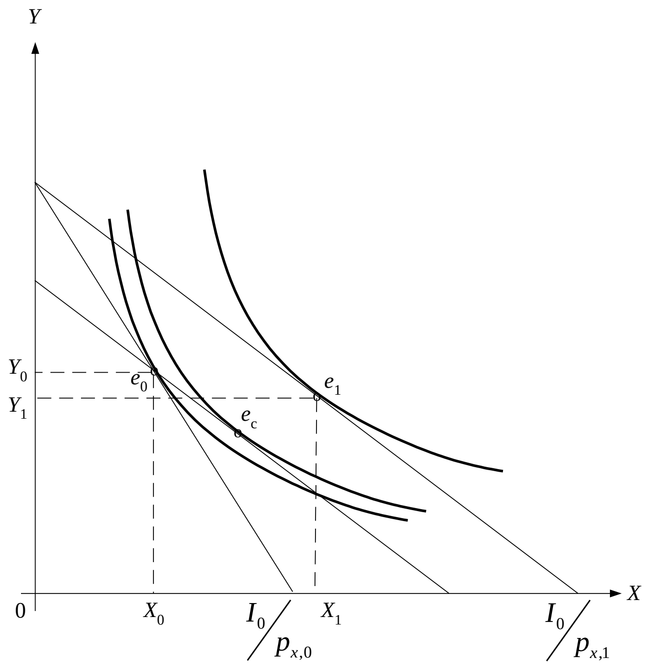
$$\frac{\Delta X}{\Delta p_x} = \left(\frac{\Delta X}{\Delta p_x}\right)_{comp} + \left(\frac{\Delta X}{\Delta p_x}\right)_{inc} .$$

Note that $\Delta I = \Delta p_x . X_0$ and therefore we can express the relationship as

$$\begin{aligned} \frac{\Delta X}{\Delta p_x} &= \left(\frac{\Delta X}{\Delta p_x}\right)_{comp} + \left(\frac{\Delta X}{\Delta p_x}\right)_{inc} \\ &= \left(\frac{\Delta X}{\Delta p_x}\right)_{comp} + \left(\frac{\Delta X}{\Delta I / X_0}\right) \\ &= \left(\frac{\Delta X}{\Delta p_x}\right)_{comp} + \left(X_0 . \frac{\Delta X}{\Delta I}\right) \end{aligned} .$$

It is conventional to express the income effect in the negative form, i.e., premultiplying by –1.

Figure 3.7 Slutsky Equation and Compensating Variation 2



4. Index Numbers and The Cost of Living

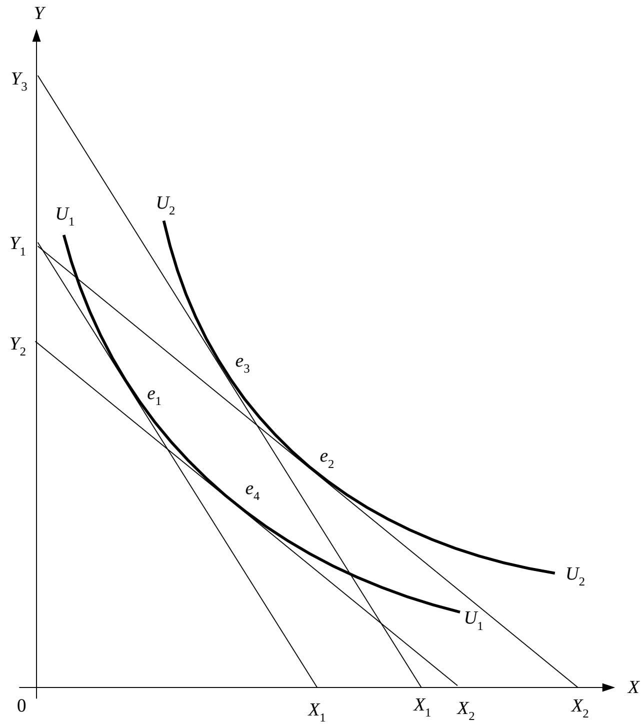
At first sight, all this Hicks and Slutsky compensating and equivalent variation theory appears to be abstract theorising for its own sake. The next two topics introduce issues that show how important these ideas are to economics. Hopefully as the course proceeds you will come to realise how crucial they are to a large proportion of applications of microeconomics.

It is not uncommon to hear people casually talking about the cost of living. Yet these theories demonstrate that the cost of living is an ambiguous concept, and in particular, that it is first necessary to specify the standard of living whose cost we wish to measure.

4.1 Compensating and Equivalent Variations and the Cost of Living

For convenience let us use the Hicks measure of real income. The income effect is a measure of how price changes impact upon the cost of purchasing a specific level of utility or the cost of achieving a specific standard of life. But which standard?

Figure 4.1 Hicks Compensating & Equivalent Variations and the Cost of Living



When making cost of living comparisons we must answer several major questions before we start. Do we wish to make the comparisons based on the base level of well-being, i.e., the initial indifference curve, or the current level of well-being, i.e., the new indifference curve? Do we wish to make the comparisons based on the base period price ratio, or the current period price ratio?

Starting at e_1 on Y_1X_1 and U_1, a fall in p_X results in a move to e_2 on Y_1X_2 and U_2. We can therefore use a compensating variation measure of the income effect, by starting from the standard of living e_1, which is $(Y_1 – Y_2)$, OR an equivalent variation measure, by starting from e_2, which is $(Y_3 – Y_1)$. In both cases the fall in p_X represents the decline in the cost of living. But the compensating variation is the reduction in income, at the new prices, to maintain utility, whereas the equivalent variation in the increase in income to maintain utility at the old prices. Notice how we have used a reduction in p_x here whereas previously we were using an increase in p_x. The effect is to reverse the diagrammatic representations of CV and EV; although the basic principal remains unchanged.

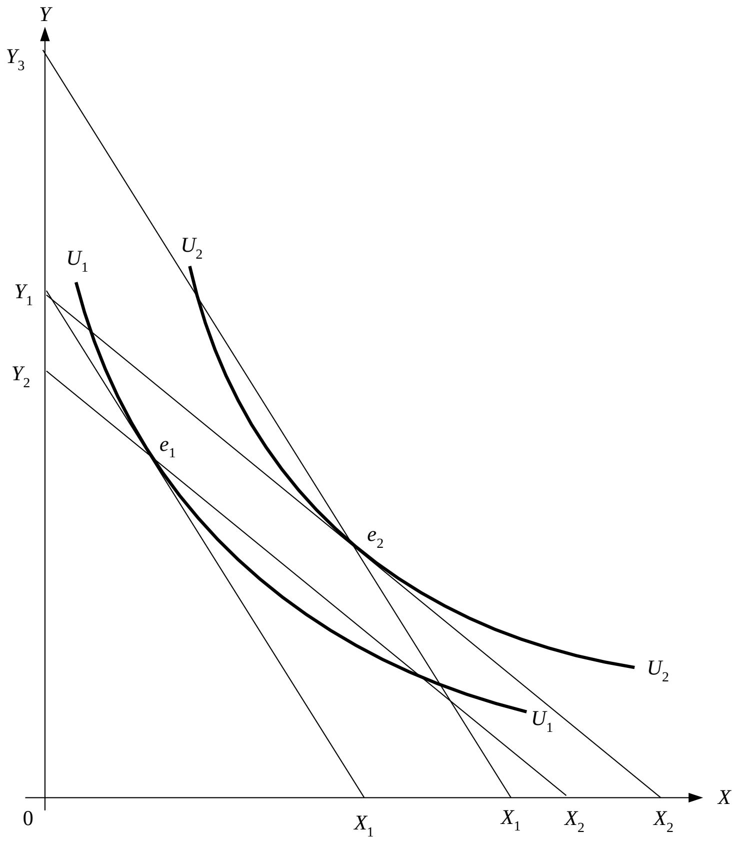
But

$$(Y_1 - Y_2) \neq (Y_3 - Y_1) \qquad \forall \Delta p_x$$

and hence the measure of the change in the cost of living is ambiguous.

Implicit to the concepts of compensating and equivalent variation are weighting systems. But, as already noted, the Hicks measures cannot be operationalised because the indifference curves cannot be observed, hence the need for the Slutsky measures. The Slutsky CV and EV measures illustrated below use the same indifference curves and budget constraints as the Hicks CV and EV case above, but, because the indifference curves U_1U_1 and U_2U_2 are unobserved the tangencies e_3 and e_4 cannot be identified. Hence rather than holding the utility level constant we hold the bundle of goods consumed constant, either at the base level, e_1, or at the current level, e_2.

Figure 4.2 Slutsky Compensating & Equivalent Variations and the Cost of Living



In fact, the compensating variation is a base weighted measure, or Laspèyres price index, i.e.,

$$L = \frac{\left(p_{X_1} X_0 + p_{Y_1} Y_0\right)}{\left(p_{X_0} X_0 + p_{Y_0} Y_0\right)}$$

and the equivalent variation is a current weighted measure, or Paasche price index, i.e.,

$$P = \frac{\left(p_{X_1} X_1 + p_{Y_1} Y_1\right)}{\left(p_{X_0} X_1 + p_{Y_0} Y_1\right)}$$

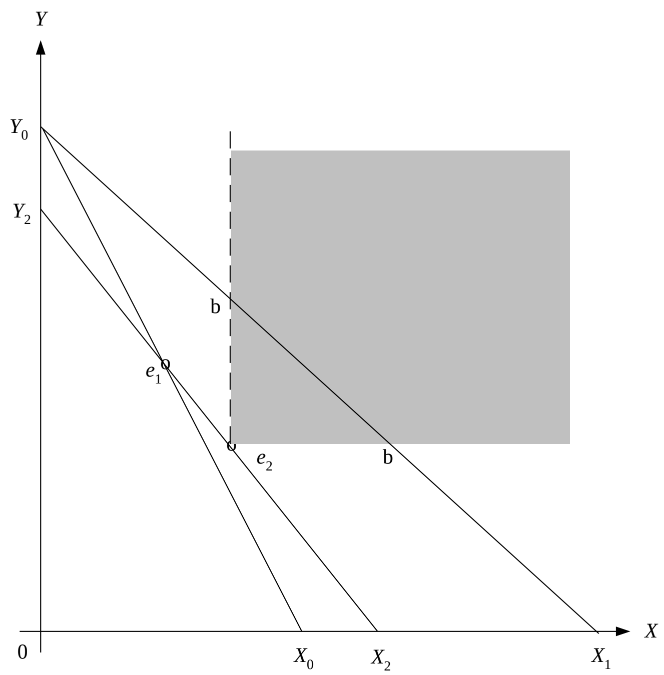
Not only will the Laspeyres and Paasche price indices (usually) differ, our theory shows that their origins in the Slutsky measures makes them approximations. Moreover, there are circumstances in which the two prices indices can have opposite signs, i.e., one indicates a rise and one a fall in the cost of living.

Since economists wish to measure things like the cost of living, average house prices etc., an awareness of what we are measuring is vital. This also provides an insight into why index number systems are considered so important in the economic literature.

4.2 Revealed Preferences and the Cost of Living

Now consider the cost of living question in the context of revealed preferences, i.e., without the use of indifference curves.

Figure 4.3 Revealed Preferences and the Cost of Living



(Y_0,X_0) is the budget constraint with the old set of prices

(Y_0,X_1) is the budget constraint with the new set of prices

(Y_2,X_2) is the Slutsky compensating variation constant real income.

We can deduce that no point to the left of e_1, the individuals initial revealed preferences, on (Y_2,X_2) will be chosen with the new set of prices since points to the left of e_1 on (Y_0,X_0) were not chosen, and such points have a higher level of utility than those on (Y_2,X_2). Thus the substitution effect will result in the selection of a point to the right of e_1 on Y_2X_2 and thus will always be negative, or zero. Assume e_2 is selected.

Now the axiom that 'more is preferred to less', indicates that any points between *bb* on (Y_0,X_1) is unambiguously preferred to e_2. Further, if *X* is an inferior good then points to the left of *bb* may be preferred. And if *Y* is inferior point to the right of *bb* may be preferred.

Revealed preference theory therefore confirms our basic results:

i) the substitution effect is never positive

ii) the income effect is positive, if *X* and *Y* are normal, but may be negative for an inferior good

iii) the income effect can never be negative for every good.

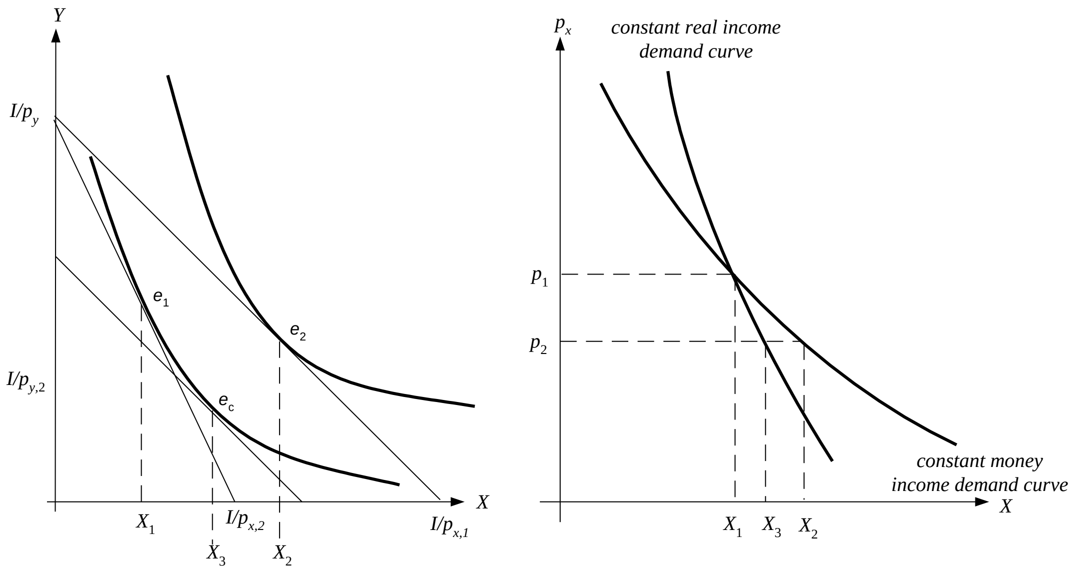
5. Real Income and Demand Curves

By now you should be realising that the *ceteris paribus* assumption we started with may be logically dubious since holding *I* and p_y constant is likely to result in a shifting demand curve when we were assuming that with changes in p_x we were constrained to movements along a demand curve.

Worse still, we might even end up concluding that by simply reducing the price of good *X* we could make everyone instantly better off, as if by magic.

Since prices and *I* are interrelated we can argue that what we want is a condition that can be held constant which combines both effects, i.e., a real income term. One candidate is a constant level of well-being/utility. Then we could conceive of a demand curve with constant real income and constant p_y. We can use this concept to derive an alternative demand curve.

Figure 5.1 Real Income and Demand Curves



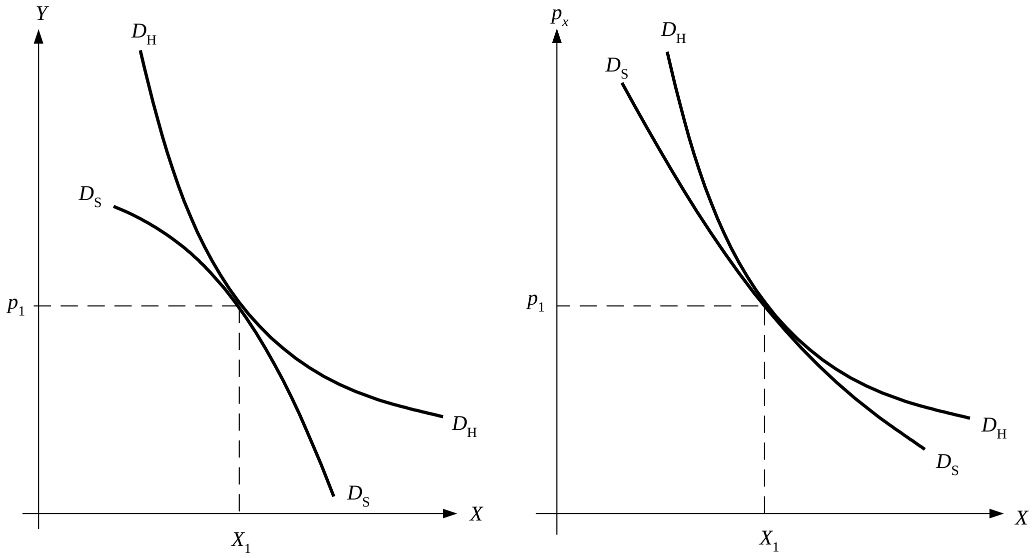
Again, we have a fall in the price of good *X* that induces positive income and substitution effects for *X*, which is a *normal* good.

Holding utility (our present real income definition) constant induces an increase in demand from X_1 to X_3, and, hence for all possible changes in p_X, allows us to derive a constant real income demand curve. In addition, we could hold money income constant and the fall in p_X would raise demand from X_1 to X_2, and allow us to identify a constant money income demand curve.

Note since *X* is a normal good the constant real income demand curve is steeper than the constant money income demand curve. The opposite would apply if *X* was an inferior good.

How does the Slutsky approximation affect this result? **You should draw your own diagrams**.

Figure 5.2 Hicks & Slutsky Constant Real Income Demand Curves



X is an INFERIOR good *X* is an NORMAL good

Note that the tangency between the two demand curves arises because for infinitesimal price changes the Hicks and Slutsky real income changes are identical.

There are differences between the demand curves derived using the compensating and equivalent variation methods. Specifically, by defining real income, *R,* we are seeking to satisfy the standard *ceteris paribus* presumption, i.e.,

$$X = f(p_x)_{\overline{R}}$$

where $\overline{R} =$ constant real income.

But

$$X = f(p_x)_{\overline{R}_C} \neq X = f(p_x)_{\overline{R}_E}$$

where $\overline{R}_C =$ compensated constant real income

$\overline{R}_E =$ equivalent constant real income

because

$$\overline{R}_C \neq \overline{R}_E$$

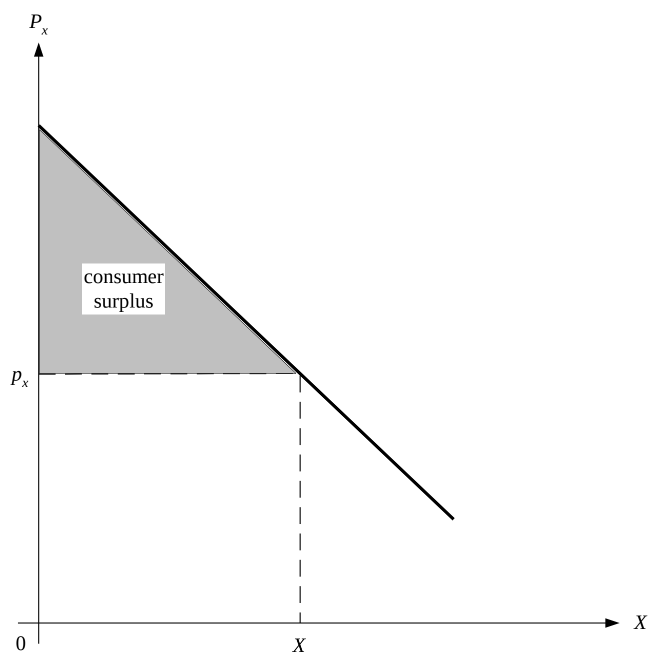
This is not a reason for concern since we would not expect the responsiveness of demand to be independent of the level of real income.

6. Consumer Surplus and Welfare

Here the concern is with *consumer surplus*, which originated with Marshall's work in the 19th century but retains a great deal of importance. It underpins cost-benefit or cost-cost analysis, all partial equilibrium welfare analysis, consumer responses to discriminatory pricing etc.

Consumer surplus can be defined as the difference between what a consumer is willing to pay for a given quantity of a good and the amount actually paid, i.e.,

Figure 6.1 Consumer Surplus and Willingness to Pay



It doesn't require a great leap of imagination to realise that deriving estimates of the real income impact of changes in prices on consumer well-being may be of interest. For example

i) if the government (of the EC) decides to increase support for EC farmers by raising food prices, how will this affect consumer welfare? (Price Policy Analysis)

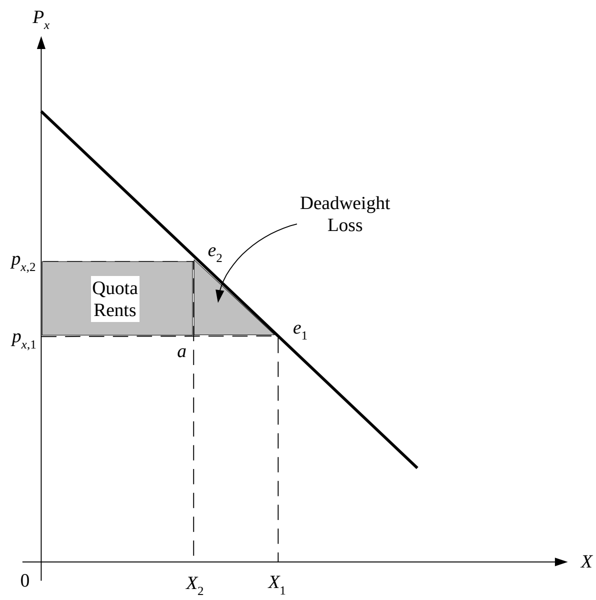
ii) if the WTO revises barriers to trade what are the welfare implications of the changes in prices that will arise? (Trade Policy Analysis)

iii) if a new road reduces transport costs, by how much will this benefit consumers? (the benefits side of CBA)

(We ignore here the problem of summing utility levels/changes across individuals).

The concept of consumer surplus, and, in particular, the change in consumer surplus, provides a very useful and powerful tool for quantitative policy analysis. Consider for instance the impact of a quota on the import of Japanese cars – the real nightmare being that we all must buy British cars. In a very simplified form of analysis we can imagine that the quota is reduced from $0X_1$ to $0X_2$ and everything else is unchanged. Given the demand curve the price of the car will rise from $p_{x,1}$ to $p_{x,2}$. Hence consumer surplus will decline by an amount equal to the area $p_{x,2}e_2e_1p_{x,1}$. However, the area $p_{x,2}e_2ap_{x,1}$ represents an increase in income to the exporters of the cars, or those who own the domestic import rights, and is therefore an economic rent that they can reap. Thus, the triangle e_2e_1a is the deadweight loss of the quota to social well-being.

Figure 6.2 Consumer Surplus and Import Quotas



But we have seen that there are some degrees of ambiguity about the precise definition of the demand curve. If we wish to use the compensating or equivalent variation real income measures, we require knowledge about the precise shape of a consumer's indifference map. Since this information is unavailable we are forced to use an approximation, just as the Slutsky measures are approximation to the Hicks measures of real income. In particular, the standard Marshallian analysis of consumer surplus presumes that the marginal utility of income is ***constant***.

If we wish to gain an 'exact' measure of consumer surplus, we would need to use the 'appropriate' compensated demand curve.