RAS and Modified RAS Techniques

Abstract

RAS is a mathematical method for reconciling data matrices that was first applied to economic data in the late 1950s and early 1960s. It requires limited computing power and retains its appeal, e.g., the procedure is used by GTAP. It is particularly useful where databases made up of reliable datapoints are available and the remaining imbalances are small. Moreover, developments of the RAS method illustrate the importance of information and the weaknesses of mathematical reconciliation methods.

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1. Introduction

Many different (mathematical) techniques have been developed and advocated for reconciling data matrices. These have been referred to as methods for balancing/estimating/projecting/forecasting the values of the cells in a matrix, with a seeming presumption that these terms are interchangeable, which can be, and often is, confusing. To avoid confusion, the approach taken in this document, and elsewhere, focuses on the derivation of Social Accounting Matrices (SAM) and refers to techniques used to adjust prior estimates of Transaction Values (TVs), i.e., cell values, to achieve data consistency (with a set of specified constraints) as reconciliation or balancing techniques, and to refer to techniques used to estimate cell values from uncertain prior estimates of cell values as estimation techniques. The approach adopted is informed by the economics of information (see Thiel, 1967).

All prior estimates of the cell/transaction values (TVs) for SAMs are uncertain, i.e., estimated with error. In part this reflects the fact that national accounts data depend on a mix of surveys, e.g., household income and expenditure and labour force surveys, censuses, e.g., censuses of manufacturing/agriculture/retail/ etc., economic reports, e.g., imports and exports and tax revenues, and imputed values, all of which are estimated with error.[1] Moreover, not all the data for a SAM can be drawn from estimates for the current year, so prior estimates of TVs may require using information from different years, which may have been derived using estimates/estimation techniques.[2] The boundary between reconciliation and estimation techniques is however opaque: in practice all estimates of the cell/transaction values (TVs) for SAMs will not fulfill a standard set of specified constraints. It is therefore a judgement call as to how far the TVs in a prior SAM must depart from the set of constraints before the process transitions from reconciliation to estimation.

One set of constraints (control totals) that is commonly imposed is the presumption that row and/or column totals are known and certain; this is implicitly based on the presumption

[1] Even before allowing for misreporting, e.g., household survey typically understate consumption (demand) of tobacco and alcohol, which requires 'adjusting' to reflect supply side information.

[2] "The most important constraint in disaggregating the tables further is the availability of data. At the present point in time, further industry and commodity detail than is published here would, for many industries, involve a substantial increase in the degree of estimation, and consequent loss of reliability." (CSO, 1973, p 7, emphasis added). There is little reason to believe that this does not continue to be the case.

that since incomes and expenditures for all accounts in a SAM must be equal then this is an objective. But, if the TVs are estimated with error, the row and column totals must be estimated with error, i.e., they are uncertain. Thus, the use of row and column totals as control totals may involve a degree of confusion over cause and effect. If ALL the TVs are estimated 'correctly', i.e., the estimates of TVs are complete, then the incomes and expenditures by all accounts will equate, i.e., consistency is a consequence of correctly estimated TVs. But even if there are estimates for ALL the TVs, the existence of uncertainty in the estimated TVs means that the incomes and expenditures by all accounts will NOT equate, i.e., INconsistency despite the estimated TVs being complete.

But consider the case where the set of estimated TVs is not complete: then, excluding exceptional circumstances, the estimated SAM will be inconsistent. In such circumstances mathematical estimation techniques can/will ensure that incomes and expenditures by all accounts will equate: the resultant SAM will be consistent but **incomplete**.

Some, or all, of the TVs in a SAM that is consistent but incomplete will be distorted, i.e., erroneous or inaccurate. If a distorted SAM is used to calibrate a whole-economy model it "invalidates the results obtained from these models" (Thorbecke, 2003, p 186).

Consequently, while mathematical techniques are useful – good tools – they bring with them 'dangers'. They can encourage those estimating matrices to economise on the expensive task of collecting data and to replace those efforts with computer algorithms.

The variants considered here are the bi-proportional or RAS method first proposed by Stone *et al* (1963), and two subsequent variants. The reasons for choosing RAS and the variants are mixed: first, in some circumstances RAS, or a variant, is a good choice; second, the technique is simple and robust and therefore provides a good learning platform; third, the technique is a good platform for illustrating some of the desirable outcomes when estimating a SAM; and fourth, it provides a simple platform for appreciating the inadequacies of consistent but incomplete SAM estimates.

2. The RAS method and Derivatives

During the 1960s a series of related reconciliation methods was proposed, Table 2.1 lists a selective sample. These techniques all involved some form of presumption that the objective was to estimate coefficients, although subsequent applications have focused on directly estimating TVs.

These techniques have been extensively examined in the literature, e.g., Bacharach (1965 and 1970), Allen (1974), Lecomber (1975), Günlük-Senesen and Bates (1988) and references cited by those authors. The relative strengths and weaknesses of these methods are now reasonably well understood, although it is not clear to what extent users understand the strengths and weaknesses

Table 2.1 Some Early Matrix Reconciliation Techniques

Proposer	Minimand	Functional Relationship
Almon (1968)	$\sum \left(x_{ij}^{*} - x_{ij} \right)^2$	$\mathbf{A}^* = \hat{\mathbf{r}}\mathbf{H} + \mathbf{A} + \mathbf{H}\hat{\mathbf{s}}$ where $\mathbf{H} = \mathbf{i}\mathbf{i}'$
Friedlander (1961)	$\sum \frac{\left(x_{ij}^{*} - x_{ij} \right)^2}{{}_0 x_{ij}}$	$\mathbf{A}^* = \hat{\mathbf{r}}\mathbf{A} + \mathbf{A} + \mathbf{A}\hat{\mathbf{s}}$
Matuszewski *et al* (1964)	$\sum \frac{\lvert x_{ij}^{*} - x_{ij} \rvert}{x_{ij}}$	None
Stone *et al* (1963)	$\sum \left(x_{ij} \log \frac{x_{ij}^{*}}{x_{ij}} \right)$	$\mathbf{A}^* = \hat{\mathbf{r}}\ \mathbf{A}\ \mathbf{s}$
Theil (1967)	$\sum \left(x_{ij} \log \frac{x_{ij}}{x_{ij}} \right)$	None

Source: adapted from Lecomber (1975)

The major justification for the choice of the RAS technique has been its attribute of preserving signs on the coefficients, while there is no clear evidence that the sign preserving characteristic of RAS is gained at the expense of reliability. However, all such matrix estimation techniques produce estimates whose reliability should be treated with care.

The exercises associated with the document are designed to demonstrate RAS and inculcate caution in users of mathematical techniques.

3. The Simple RAS Method

The simple RAS method, as originally formulated by Stone *et al* (1963), was an attempt to devise a technique by which the cell values of a matrix could be adjusted to conform to known row and column totals (controls) by some form of loss minimisation technique. This was achieved by the iterative estimation of two vectors **r** and **s** that operate on a matrix of best estimates to adjust the values of the cells such that the resultant matrix conforms with known control totals. Early applications concentrated on inter-industry/input-out matrices.

Mathematically

$$\mathbf{X}^* = \hat{\mathbf{r}}\mathbf{X}\hat{\mathbf{s}} \tag{1}$$

subject to

$$\mathbf{X}^*\mathbf{i} = \mathbf{u} \qquad \textit{and} \qquad \left(\mathbf{X}^*\right)'\mathbf{i} = \mathbf{v} \tag{2}$$

where **X** is the matrix of prior estimates, $\mathbf{X}^*$ is the outturn matrix, **x** is the known vector of gross industrial output, $\mathbf{X}^*$ is the estimated inter-industry flow matrix, and **u** and **v** are the known row and column control vectors respectively.[3] Therefore, the typical element of $\mathbf{X}^*$ is

$$x_{ij}^* = r_i x_{ij} s_j \tag{3}$$

which Bacharach (1965 and 1970)[4] showed to result from the minimisation of

$$\sum_{j=1}^{n} x_{ij}^* . \log \frac{x_{ij}^*}{x_{ij}} \tag{4}$$

subject to

$$\sum_{j=1}^{n} x_{ij}^* = u_i \qquad \text{and} \qquad \sum_{i=1}^{n} x_{ij}^* = v_j \ . \tag{5}$$

3 If the problem is formulated as a coefficient matrix then Eqn 1 may be written as $\mathbf{A}^* = \hat{\mathbf{r}}\mathbf{A}\hat{\mathbf{s}}$ and Eqn 2 as $\left(\mathbf{A}^*\hat{\mathbf{x}}\right)\mathbf{i} = \mathbf{X}^*\mathbf{i} = \mathbf{u}$ and $\left(\mathbf{A}^*\hat{\mathbf{x}}\right)'\mathbf{i} = \left(\mathbf{X}^*\right)'\mathbf{i} = \mathbf{v}$, where $\mathbf{A}^*$ is the solution matrix of coefficients, **x** is the known vector incomes, and $\mathbf{X}^*$ is the estimated matrix in TVs.

4 J.M.Bates (personal communication) confirms that this proof was first derived in 1964.

Solution Methods

All solution methods assume that the solution values must be consistent with a set of row and column control totals that are known and certain. The solution algorithms for simple RAS traditionally used iterative methods to derive estimates of the **r** and **s** vectors required to adjust a prior set of estimates into a reconciled set of estimates. The method is reminiscent of the Newton method for finding square roots: calculate a vector such that when every entry in a row of the prior is multiplied by the respective element in the vector produces an updated prior that summed across the columns is equal to the row control totals; then calculate a vector such that when every entry in a column of the updated prior is multiplied by the respective element in the vector is equal to the column control totals; continue iterating until the updated prior is consistent with the row **and** column totals.

More recent applications solve the simple RAS method using on optimisation method that minimises the optimand in Eqn 4. This introduces a potential problem through the need to take logarithms of values that can be negative.

It has been suggested (Günlük-Şenesen and Bates, 1988, p 478) that four criteria are appropriate for evaluating the suitability of mathematical methods:

1. the 'closeness' of the estimated matrix with the initial matrix;
2. the preservation of zeros;
3. the preservation of signs; and
4. consistency with extraneous information.

Given these criteria some of the properties of the RAS formulation can be evaluated in theory. The optimand is a weighted sum of the logs of the ratios of the prior, a_{ij}, to the solution, a_{ij}^*, values where the weights are the solution values, i.e., a logarithmic loss function that could also be expressed a weighted distance function. Trivially, the value of the optimand will be related to the divergences between the prior and solution values for transactions, from which it can be surmised that if more information can be included in the prior the more reliable will be the solution values. It is therefore unsurprising that national accounts experts have long emphasised the importance of the completeness and accuracy of the prior estimates of transaction values (TVs). The supporting exercises demonstrate the importance of the prior estimates.

The RAS method preserves zero values: if the prior estimates for a cell, a_{ij}, are zero then the solution value, a_{ij}^*, will be zero (see Eqn 3). The preservation of signs is more complex. The traditional iteration method has been found to be capable of finding solutions even if there are some negative transactions, typically subsidies (negative taxes). But the optimisation methods have problems with negative transactions because the log of a negative is undefined. There are however two simple solutions first, transpose the negative values in the prior and adjust the control totals accordingly, and/or second fix the negative transactions so that they can be removed from the optimisation problem (see Modified RAS below).

Accordingly, there may be grounds to suggest RAS may be an appropriate option for rendering a matrix consistent.

4. Modified RAS

One option to reduce the loss of information with RAS is to add information in addition to the row and column control totals. A modification to the RAS method proposed by Allen (1974) provides a means by which additional contemporaneous information can be added. Define **K** as the matrix of known contemporaneous values with all other cells set equal to zero then the RAS method can be reformulated as

$$\mathbf{A}^* = \mathbf{K} + \hat{\mathbf{r}}\left(\mathbf{A} - \mathbf{K}\right)\hat{\mathbf{s}} \tag{7}$$

or in terms of the typical element

$$a_{ij}^* = k_{ij} + r_i\left(a_{ij} - k_{ij}\right)s_j \tag{8}$$

where both are subject to the constraints in 2 and 5 respectively.

This method ensures the preservation of signs (assuming one of the previously mentioned options is used) and zeros. It is also trivial to demonstrate that the solution values will be closer to the prior values for all of the five measures identified above, i.e., for each of the cells in **K** the value of $\left(x_{ij}^* - x_{ij}\right)$ will be zero, which will reduce the value of the measure of closeness.

5. RAS as an Optimisation Problem in GAMS

RAS can be readily formulated in GAMS as an optimisation problem as four blocks of equations. The first defines the solution as a square matrix that is the product of $\mathbf{A}^* = \hat{\mathbf{r}}\mathbf{A}\hat{\mathbf{s}}$

$$SAM\_RAS_{ss,ssp} = R_{ss} * S_{ssp} * SAM\_prior_{ss,ssp} \quad ()$$

subject to column control totals $\left(\mathbf{X}^*\mathbf{i} = \mathbf{u}\right)$

$$col\_con0_{ss} = \sum_{ss} SAM\_RAS_{ss,ssp} \quad ()$$

subject to row control totals $\left(\left(\mathbf{X}^*\right)'\mathbf{i} = \mathbf{v}\right)$

$$row\_con0_{ss} = \sum_{ssp} SAM\_RAS_{ss,ssp} \,. \quad ()$$

The objective function is $\sum_{j=1}^{n} a_{ij}^* . \log \frac{a_{ij}^*}{a_{ij}}$, which can be written as

$$OB = \sum_{ss,ssp} SAM\_RAS_{ss,ssp} * \left[\log\left(SAM\_RAS_{ss,ssp} + \beta\right) - \log\left(SAM\_prior_{ss,ssp} + \beta\right)\right] \,. \quad ()$$

The inclusion of the parameter beta ensures that the programme does not try to derive the log of zero.

The programme is solved by minimising the value of the objective function (OB).

In GAMS code this is written as

```
BIPROP(ss,ssp)..   SAM_ras(ss,ssp) =E= R(ss)*S(ssp)*SAM_prior(ss,ssp) ;

 CCONT(ssp)..   col_con0(ssp) =E= SUM(ss, SAM_ras(ss,ssp)) ;

 RCONT(ss)..    row_con0(ss) =E=  SUM(ssp, SAM_ras(ss,ssp)) ;

 OBJ..       OB =E= SUM{(ss,ssp)$SAM_prior(ss,ssp), SAM_ras(ss,ssp)*
                        [log(SAM_ras(ss,ssp) + beta)
                       - log(SAM_prior(ss,ssp) + beta)]} ;
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6. RAS with Known Subtotals

The solution values generated by RAS cannot be certain to add up to some external aggregates, e.q., GDP. Ensuring the solution values are consistent with certain exogenously defined values is marginally more complex. Günlük-Şenesen and Bates (1988, p 480) demonstrate that this issue can be resolved by adding an additional multiplier, q, that is applied to those TVs that sum to some known exogenous value, i.e.,

$$a_{ij}^{*} = r_i a_{ij} s_j q \quad (6)$$

An alternative method would be to redefine selected accounts so that the row and column totals complied with the exogenous information. For instance, with GDP defined as $C + G + I + X - M$, an account for GDP can be defined – summing $C + G + I + X - M$ - and the totals fixed so that C, G, I, X and M adjust to satisfy the condition. Similarly, if, for example, C is known an account can be defined for household consumption expenditure separate from taxes and saving with the account total fixed exogenously.

7. RAS with Uncertain Control Totals

However, both the RAS procedure and Allen's modification assume that the control totals are known with certainty which may not always be the case, e.g., changes in industry classification schemes may mean that the control total are not known with certainty. Noting the requirement for the preservation of the social accounting identity, i.e.,

$$\mathbf{i}'\mathbf{u} = \mathbf{i}'\mathbf{v} \qquad ()$$

Allen and Lecomber (1975) addressed the problem of potential uncertainty in the control totals. By attaching errors, $\mathbf{e}_u$ and $\mathbf{e}_v$, to the row and column controls respectively, then estimates of **u**, $\mathbf{u}^*$, **v**, and $\mathbf{v}^*$ can be derived from

$$\mathbf{u}^* = \mathbf{u} + \mathbf{e}_u - \hat{\mathbf{r}}\mathbf{e}_u \qquad ()$$

And

$$\mathbf{v}^* = \mathbf{v} + \mathbf{e}_v - \hat{\mathbf{s}}\mathbf{e}_v \qquad ()$$

where **r** and **s** are derived as before. This does not in itself preserve the accounting identity, but if $\mathbf{e}_u$ and $\mathbf{e}_v$ are incorporated as an additional row and column of the initial matrix, and $\mathbf{i}'\mathbf{e}_u$ and $\mathbf{i}'\mathbf{e}_v$ as additional elements in the row and column control vectors then the accounting identity will be preserved.

8. Applications of Coefficient Projection Procedures

Three applications of coefficient estimating procedures most pertinent to this study are worthy of brief consideration. First, the RAS procedure has been used in the preparation of the UK IO tables to achieve "complete balance in every input-output group" (Lynch, 1990)5. The reason why some estimating technique must be used is easily understood. The degree of reliability of the data relating to inter-industry transactions is inevitably somewhat less than that of the control totals, thus when flows tables are compiled it is likely that a summation of the recorded transactions will differ from the controls. In this instance RAS can be readily applied to the 'best estimates' of the actual transactions so as ensure the resultant matrices are balanced.

Second, an estimating procedure may be used to derive coefficient estimates for a period for which an IO table already exists, but which had been compiled using a different industry classification scheme. One way of attempting to cover these periods would be to use an estimating procedure to derive estimates of transactions for the missing years based on the definitions used in an adjacent period. Since changes in classification schemes are typically partial, it may be possible to use this characteristic as the source of certain information. Using Allen's technique, the elements of **K** would be those elements for which both parties to transactions are from groups for which the definitions remained constant, the best estimates for the uncertain elements being derived from the base period matrices. This does have a drawback. Whereas Allen's test of his proposed modification assumed that the positive cells in **K** would be the "major" coefficients in this case only some of the cells of (**A** - **K**) would be 'major' coefficients. This means that Allen's test cannot be unambiguously invoked as *a priori* justification for the choice of technique. The reason is the propensity of the RAS procedure to impose most of the adjustment for each row and column of the matrix (**A** - **K**) on the cells in which the 'major' coefficients are located. Despite this weakness the use of contemporaneous IO data has much to recommend it, while additional exogenous information would clearly enhance the accuracy of the estimates. However, it is likely that the changes in classification

5 It appears that an estimating procedure has been used at least since the compilation of the 1968 tables. "The most important constraint in disaggregating the tables further is the availability of data. At the present point in time, further industry and commodity detail than is published here would, for many industries, involve a substantial increase in the degree of estimation, and consequent loss of reliability." (CSO, 1973, p 7, emphasis added).

render the control totals uncertain, which suggests the use of a combination of Allen's and Allen's and Lecomber's procedures.

The third possible application relates to the provision of estimated IO tables in years for which full IO tables were/are unavailable. Two scenarios can be envisaged for the provision of such tables. First, for periods since the last full IO tables were available[6]. And second for periods between full tables, in both instances the obvious choice of procedure, of those considered here, would be Allen's supplemented by Allen and Lecomber's for periods over which there have been classification changes. For the first scenario the control totals are already effectively gathered for the national accounts, thus the only additional data requirement would be the 'major' inter-industry transactions, for which much of the data is probably already collected. Similarly, this information could also be used to update the estimated tables after each new set of full IO tables had been prepared, using the two full sets of tables to enhance the best estimates. While such IO tables may lack the accuracy of full sets they would allow far greater use of the IO methodology in the empirical analysis of economic relationships.

Applications

The most common use of simple RAS has been for the estimation of a matrix for a year from the matrix of another year when the control totals are known with 'certainty'.

There have been numerous studies that have sought to evaluate the appropriateness of RAS as a technique for 'projecting' TVs in matrices, e.g., the Lecomber (1975), Lynch (1979), Günlük-Şenesen and Bates (1988), Schneider and Zenios (1990) and Britz (2021). Typically, this will be expressed as some measure of the 'closeness' of the solution values to the prior values: Günlük-Şenesen and Bates (1988) observe that no single measure of 'closeness' and identify five 'measures'[7]

1. mean absolute deviation (MAD), which is $\left(\frac{1}{n}\right)\sum_{i,j}\left|x_{ij}^{*}-x_{ij}\right|$, where n is the number of elements in the table,

[6] However, Lynch (1979) has argued "that little should be expected of it as a means of forecasting absorption matrices over a period as long as five years" (p 283).

[7] The authors also identify correlation coefficient (COR) between x_{ij}^{*} and x_{ij}

2. goodness of fit (GOF), equal to $\left(\frac{1}{n}\right)\sum_{i,j}\left|\frac{\left(x_{ij}^{*}-x_{ij}\right)^{2}}{x_{ij}^{*}}\right|$;

3. mean squared deviations (MSD), equal to $\left(\frac{1}{n}\right)\sum_{i,j}\left(x_{ij}^{*}-x_{ij}\right)^{2}$;

4. mean absolute proportionate error (MAPE), $\left(\frac{1}{n}\right)\sum_{i,j}\left|\frac{\left(x_{ij}^{*}-x_{ij}\right)}{x_{ij}^{*}}\right|$; and;

5. information change (INFO), equal to $\sum_{i,j}^{n} x_{ij}^{*}.\log\frac{x_{ij}^{*}}{x_{ij}}$.

The efforts to evaluate the performance of RAS, in various forms, and other mathematical techniques have typically proceeded by using data from a period to develop a prior that is then used to estimate a known matrix for a subsequent period and then to evaluate performance. Or to start with a known matrix that is perturbed to derive a prior matrix that is then used to estimate a matrix that is compared with the known matrix.

All known studies come to a similar conclusion: the more information contained in the prior matrix the more likely the solution matrix is to be reliable. More generally, it is probable that no method is universally ideal (see Günlük-Senesen and Bates, 1988, p 489) and the choice will depend on circumstances. Two conclusions are perhaps most relevant:

(i) "that little should be expected of it as a means of forecasting absorption matrices over a period as long as five years" (Lynch, 1979, p 283)[8], and

(ii) "The search for, and acquisition of, additional information may often be more important than the choice made of adjustment method" (Günlük-Senesen and Bates, 1988, p 489)[9].

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8 Lynch's study was the first known study that cast serious doubt on using RAS to 'update' matrices because the method does no account well for structural changes.

9 This conclusion reinforced earlier arguments that there was no ideal mathematical method to substitute for enhancing the information content of the prior matrix/SAM.

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