A Static Applied General Equilibrium Model:

Technical Documentation

STAGE_DEV Version 2: August 2016[1]

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1 The STAGE model is subject to ongoing developments; this version of the technical document contains details of developments up to the given date. Earlier versions of this model were named differently; the PROVIDE version is the latest of the earlier versions for which documentation is readily available (PROVIDE, 2005).
Various collaborators have contributed to the development of this model. See Appendix 1 on the model's genealogy for details.

2 A number of the behavioural relationships in the STAGE_DEV were first given concrete form in the PhD of Emerta Aragie (2015). In the STAGE_DEV variant the specific formulations of these behavioural relationships have been further developed to make them more robust.

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Introduction

This document provides a description of the comparative static version of STAGE_DEV single-country computable general equilibrium (CGE) model, that is a variant/development of the STAGE 2 single country CGE model. Recursive dynamic applications of the STAGE family of CGE models all start from the respective comparative static models, by exploiting the LOOP facility provided by GAMS (General Algebraic Modelling System[3]), so that the recursive dynamic applications operate as series of comparative static simulations where the 'dynamic' updates are implemented between each comparative static simulation. Thus, an in-depth understanding of the comparative static versions of the model is essential before progressing to the recursive dynamic versions.

This model is characterised by several distinctive features. First, the model allows for a generalised treatment of trade relationships by incorporating provisions for non-traded exports and imports, i.e., commodities that are neither imported nor exported, competitive imports, i.e., commodities that are imported and domestically produced, non-competitive imports, i.e., commodities that are imported but not domestically produced, commodities that are exported and consumed domestically and commodities that are exported but not consumed domestically. Second, the model allows the relaxation of the small country assumption for exported commodities that do not face perfectly elastic demand on the world market. Third, the model allows for modeling of multi-product activities using various assumptions; fixed proportions of commodity outputs by activities with commodities differentiated by the activities that produce them, varying output mixes by activities in response to changes in the (basic) prices of commodities, and domestically produced commodities that are differentiated by source activity or are homogeneous, i.e., undifferentiated by source activity. Hence the numbers of commodity and activity accounts are not necessarily the same. Fourth, the (value added) production technologies can be specified as nested Constant Elasticity of Substitution (CES). Fifth, trade and transport margins between factory and dock gate and the consumer are levied on domestic consumption. Sixth, consumption expenditure by each representative household group (RHG) is represented by nested CES and Stone-Geary utility functions. Seventh, household consumption commodities include 'leisure' where the 'leisure' consumed by each RHG can only be produced by labour factors owned by the respective RHG; this

[3] See www.gams.com for details about GAMS.

introduces a labour-leisure trade-off into the model. Eighth, the model is calibrated using two matrices detailing the factor **use** by activity and the factor ownership by institution. Ninth, the functional distribution of income is endogenously determined through the specification of the ownership (domestic and foreign) of factors used within the economy being defined as a series of variables. Tenth, factor markets are segmented such that each category of factor, e.g., skilled labour, can be differentiated by the activity, or group of activities, that employs that category of factor. Thereafter the degree (elasticity) of mobility of each factor type between different activities can be varied between zero (perfectly immobile) and infinity (perfectly mobile) depending on model settings selected by the user and changes in relative wage rates. And eleventh, the model encompasses the possibility for RHGs to migrate from one RHG to another and to take the factors owned by the migrating RHG to its new RHG. The degree (elasticity) of migration of each RHG between different RHGs can be varied between zero (perfectly immobile) and near infinity (nearly perfectly mobile) depending on model settings selected by the user and changes in relative household incomes. The interplay of factor mobility and RHG migration, together with employment changes and the labour-leisure trade-offs, changes the patterns of factor ownership by each RHG and hence changes the functional distribution of income.

A guiding principle underpinning the STAGE family of models, inherited from the work by Sherman Robinson and various colleagues, is that the models should be agnostic with respect to macroeconomic closure and market clearing conditions, i.e., the models should be coded so as to allow the user substantial degrees of freedom to impose their own views on how an economy operates. The models also include a substantial number of variables included so as to simplify the coding of adjustments needed for experiments; by and large these adjustment instruments use a standard formulation.

The developments embedded in STAGE_DEV draw on previous research on factor mobility, by McDonald and Thierfelder (2007), Polaski *et al*., (2009) and Flaig *et al*., (2013), and extend the treatment of factor mobility so as to endogenise the functional distribution of income. The addition of household migration functions that address directly the issues by the possibility that households of a particular type may transition into another particular type, e.g., rural households may transition into urban households. The code used for this development is similar to that used for factor mobility functions although the incentives to migrate – changes in relative institutional incomes – are different. The code also ensures that as households migrate the functional distribution of income changes and, as such, is a development of ideas include in Ariagie (2015).

The model is designed for calibration using a reduced form of a Social Accounting Matrix (SAM) that broadly conforms to the UN System of National Accounts (SNA). Table 1 contains a macro SAM in which the active sub matrices are identified by *X* and the inactive sub matrices are identified by 0. In general, the model will run for any SAM that does not contain information in the inactive sub matrices and conforms to the rules of a SAM.[4] In some cases a SAM might contain payments from and to both transacting parties, in which case recording the transactions as net payments between the parties will render the SAM consistent with the structure laid out in Table 1.

The most notable differences between this SAM and one consistent with the SNA are:

1) The SAM is assumed to contain only a single ‘stage’ of income distribution. However, transforming the functional distribution of income using apportionment (see Pyatt, 1989) will render the SAM consistent.
2) A series of tax accounts are identified (see below for details), each of which relates to specific tax instruments. Thereafter a consolidated government account is used to bring together the different forms of tax revenue and to record government expenditures. These adjustments do not change the information content of the SAM, but they do simplify the modeling process. However, they do have the consequence of creating a series of reserved names that are required for the operation of the model.[5]

The model contains a section of code, immediately after the data have been read in, that resolves a number of common ‘issues’ encountered with SAM databases by transforming the SAM so that it is consistent with the model structure without any marked loss of information. Specifically, all transactions between an account with itself are eliminated by setting the appropriate cells in the SAM equal to zero. Second, some transfers from domestic institutions to the Rest of the World and between the Rest of the World and domestic institutions are treated net as transfers to the Rest of the World and domestic institutions, by transposing and changing the sign of the payments. And third, some transfers between domestic institutions

4 If users have a SAM that does not run with no information in inactive sub matrices the authors would appreciate a copy of the SAM so as to further generalise the model.

5 These and other reserved names are specified below as part of the description of the model.

and the government are treated as net and as payments from government to the respective institution. Since these adjustments change the account totals, which are used in calibration, the account totals are recalculated within the model.

Table 1 Macro SAM for the Standard Model

	Commodities	Activities	Factors	Households	Enterprises	Government	Capital Accounts	RoW
Commodities	0	X	0	X	X	X	X	X
Activities	X	0	0	0	0	0	0	0
Factors	0	X	0	0	0	0	0	X
Households	0	0	X	0	X	X	0	X
Enterprises	0	0	X	0	0	X	0	X
Government	X	X	X	X	X	0	0	X
Capital Accounts	0	0	X	X	X	X	0	X
RoW	X	0	X	X	X	X	X	0
Total	X	X	X	X	X	X	X	X

In addition to the SAM, which records transactions in value terms, two additional databases are used by the model. The first are two satellite accounts that record the ‘quantities’ of primary inputs used by each activity and the quantities of factors owned by each institution. If such quantity data are not available, then the entries in the factor use and ownership matrices are the same as those in the corresponding sub matrices of the SAM. The second series of additional data are the elasticities of substitution for imports and exports relative to domestic commodities, the elasticities of substitution for the CES production functions, the income elasticities of demand for the linear expenditure system and the Frisch (marginal utility of income) parameters for each household, and factor mobility and household migration elasticities.

All the data are accessed by the model from data recorded in Excel and GDX (GAMS data exchange) file. All the data recorded in Excel are converted into GDX format as part of the model.

A key design principle of the model is that it is a ‘template’ model. In this case the term ‘template’ is defined as meaning that the model has been compiled with the expectation that users of the model are likely to, and/or should, make changes to the model so as to customise the model to the specific circumstances of the economy being studied and/or the policy issues be simulated.

The developers of and contributors to this model are supportive of users who extend the model and hope that such users will share their efforts with other users.

The Computable General Equilibrium Model

The model is a member of the class of single country CGE models that are descendants of the approach to CGE modeling described by Dervis *et al*., (1982). More specifically, the implementation of this model, using the GAMS (General Algebraic Modeling System) software, is a direct descendant and development of models devised in the late 1980s and early 1990s, particularly those models reported by Robinson *et al*., (1990), Kilkenny (1991) and Devarajan *et al*., (1994). The model is a SAM based CGE model, wherein the SAM serves to identify the agents in the economy and provides the transactions database with which the model is calibrated. Since the model is SAM based it contains the important assumption of the law of one price, i.e., prices are common across the rows of the SAM.[6] The SAM also serves an important organisational role since the groups of agents identified by the SAM structure are also used to define sub-matrices of the SAM for which behavioural relationships need to be defined. As such the modeling approach has been influenced by the 'SAM Approach to Modeling' (Pyatt, 1989; Drud *et al*., 1986).

The description of the model proceeds in five stages. The first stage is the identification of the behavioural relationships; these are defined by reference to the sub matrices of the SAM within which the associated transactions are recorded. The second stage is definitional, and involves the identification of the components of the transactions recorded in the SAM, while giving more substance to the behavioural relationships, especially with those governing inter-institutional transactions, and in the process defining the notation. The third stage uses figures to explain the nature of the price and quantity systems for commodity and activity accounts that are embodied within the model. In the fourth stage an algebraic statement of the

[6] The one apparent exception to this is for exports. However, the model implicitly creates a separate set of export commodity accounts and thereby preserves the 'law of one price', hence the SAM representation in the text is actually a somewhat condensed version of the SAM used in the model (see McDonald, 2007).

model is provided; the model's equations are summarised in a table that also provides (generic) counts of the model's equations and variables. A full listing of the parameters and variables contained within the model are located in Appendix 1.[7] Finally, in the fifth stage, there is a discussion of the default and optional macroeconomic closure and market clearing rules available within the model.

Behavioural Relationships

While the accounts of the SAM determine the agents that can be included within the model, and the transactions recorded in the SAM identify the transactions that took place, the model is defined by the behavioural relationships. The behavioural relationships in this model are a mix of non-linear and linear relationships that govern how the model's agents will respond to exogenously determined changes in the model's parameters and/or variables. Table 2 summarises these behavioural relationships by reference to the sub matrices of the SAM.

Households are assumed to choose the bundles of commodities they consume so as to maximise utility subject to a utility functions that are nested CES and Stone-Geary, where the arguments in the Stone-Geary functions are, typically, aggregates. For a developing country a Stone-Geary function may be generally preferable because it allows for subsistence consumption expenditures, which is an arguably realistic assumption when there are substantial numbers of very poor consumers.[8] But the assumption that households define subsistence consumption requirements at the level of the individual commodity, however disaggregated the commodity accounts are in the data/model, is both highly restrictive and unrealistic. It is realistic however to assume that households (of all sorts) will have subsistence consumption requirements across 'broad' commodity groups, e.g., food, while within those commodity groups households may elect to substitute between commodities (natural commodities) of the 'broad' groups, e.g., between different grains (wheat, rice, etc.,) and between vegetable and meat commodities. Consequently, this model includes nested CES and LES utility functions that at the top/LES level involve substitution between 'broad' commodity groups, subject to subsistence consumption constraints on these 'broad' groups, while at the lower level households are willing, and able, to substitute between the component

7 The model includes specifications for transactions that were zero in the SAM. This is an important component of the model. It permits the implementation of policy experiments with exogenously imposed changes that impact upon transactions that were zero in the base period.

8 A Stone-Geary function reduces to a Cobb-Douglas function given appropriate specification of the parameters.

commodities that make up the ‘broad’ commodity groups. The model includes facilities for the user to define the ‘broad’ commodity groups and the components of each group, thus the user could define ‘food’ as a broad commodity group that is a composite formed from different natural food commodities, e.g., meat, grains, fruits, etc. Another alternative would be to distinguish between commodities that are purchased on the market and those produced at home to produce a composite commodity consumed by households, e.g., composite wheat made up of market and home produced wheat where the types of wheat can be distinguished by time and/or place and/or variety.

The households choose their consumption bundles of natural commodities from a set of ‘composite’ commodities that are aggregates of domestically produced and imported commodities. These ‘composite’ commodities are formed as Constant Elasticity of Substitution (CES) aggregates that embody the presumption that domestically produced and imported commodities are imperfect substitutes. The optimal ratios of imported and domestic commodities are determined by the relative prices of the imported and domestic commodities. This is the so-called Armington ‘insight’ (Armington, 1969), which allows for product differentiation via the assumption of imperfect substitution (see Devarajan *et al*., 1994). The assumption has the advantage of rendering the model practical by avoiding the extreme specialisation and price fluctuations associated with other trade assumptions, e.g., the Salter/Swan or Australian model. In this model the country is assumed to be a price taker for all imported commodities.

Table 2 Behavioural Relationships for the Standard Model

	Commodities	Activities	Factors	Households	Enterprises	Government	Capital	RoW	Total	Prices
Commodities	0	Leontief Input-Output Coefficients	0	Nested CES and Stone-Geary Utility Functions	Fixed in Real Terms	Fixed in Real Terms and Export Taxes	Fixed Shares of Savings	Commodity Exports	Commodity Demand	Consumer Commodity Price Prices for Exports
Activities	Domestic Production	0	0	0	0	0	0	0	Constant Elasticity of Substitution Production Functions	
Factors	0	Factor Demands (CES)	0	0	0	0	0	Factor Income from RoW	Factor Income	
Households	0	0	Variable Shares of Factor Income	Fixed shares of income	Fixed Shares of Dividends	Fixed (Real) Transfers	0	Remittances	Household Income	
Enterprises	0	0	Variable Shares of Factor Income	0	0	Fixed (Real) Transfers	0	Transfers	Enterprise Iincome	
Government	Tariff Revenue Domestic Product Taxes	Indirect Taxes on Activities	Variable Shares of Factor Income Direct Taxes on Factor Income	Direct Taxes on Household Income	Fixed Shares of Dividends Direct Taxes on Enterprise Income	0	0	Transfers	Government Income	
Capital	0	0	Depreciation	Household Savings	Enterprise Savings	Government Savings (Residual)	0	Current Account 'Deficit'	Total Savings	
Rest of World	Commodity Imports	0	Variable Shares of Factor Income	0	0	0	0	0	Total 'Expenditure' Abroad	
Total	Commodity Supply	Activity Input	Factor Expenditure	Household Expenditure	Enterprise Expenditure	Government Expenditure	Total Investment	Total 'Income' from Abroad		
	Producer Commodity Prices Domestic and World Prices for Imports	Value Added Prices								

Domestic production uses a three-stage production process. In the first stage aggregate intermediate and aggregate value added (primary inputs) are combined using either CES or Leontief technologies. At the top level aggregate intermediate inputs are combined with aggregate primary inputs to generate the outputs of activities; if a CES specification is chosen then the proportion of aggregate intermediates and aggregate primary inputs vary with the (composite) prices of the aggregates, while if a Leontief specification is chosen then aggregate intermediates and aggregate primary inputs are in fixed proportions. At the second level aggregate intermediate inputs are generated using Leontief technology so that intermediate input demands are in fixed proportions relative to aggregate intermediates inputs of each activity. At the second level natural and aggregate primary inputs are combined to form aggregate value added using CES technologies, with the optimal ratios of primary inputs being determined by relative factor prices. Finally, at the third stage natural primary inputs are combined, using CES technologies, to produce aggregate primary inputs; typically, this level involves the aggregation of different types of labour to form an aggregate labour input to the second level.

The activities are defined as multi-product activities that produce combinations of commodity outputs. The model allows for a range of different assumptions governing the output mix produced by each activity. The first is a pure by-product assumption whereby the proportionate combinations of commodity outputs produced by each activity/industry remain constant; hence for any given vector of commodities demanded there is a unique vector of activity outputs that must be produced.[9] Alternatively, activities can adjust their output mixes in the response to changes in the relative (basic) prices of domestically produced commodities using CET technologies. The user can assign some activities to each of these two alternatives. The total supply of domestically produced commodities across activities can be defined in two ways: first the commodities can be differentiated by domestic activity and then aggregated using CES technologies or the commodities can be assumed to be homogenous – this latter assumption requires that the users configures the model so as to define the scale of output of the homogenous commodities from different activities.

The vector of commodities demanded is determined by the domestic demand for domestically produced commodities and export demand for domestically produced

[9] This specification is found in the IFPRI standard model (Lofgren *et al*., 2001).

commodities. Using the assumption of imperfect transformation between domestic demand and export demand, in the form of a Constant Elasticity of Transformation (CET) function, the optimal distribution of domestically produced commodities between the domestic and export markets is determined by the relative prices on the alternative markets. The model can be specified as a small country, i.e., price taker, on all export markets, or selected export commodities can be deemed to face downward sloping export demand functions, i.e., a large country assumption.

The model includes code for the endogenous determination of the functional distribution of income. Specifically factor supplies are defined by reference to their ownership by different domestic (households, enterprises and government) and foreign institutions. In its simplest form this formulation defines the quantities of factors supplied by each institution as fixed and equal to the quantities owned by each institution: this requires that factor and institutions cannot transition between categories and that there is full employment and hence the functional distribution of income is in fixed proportions. However, in this model variant the quantities of factors supplied and owned by each institution can change and hence the functional distribution of income must be defined by variables. The most common application of this, in comparative static applications, is in the context unemployment whereby some institutions may be able to supply more labour; if this happens then the share of labour supplied by each institution/household may change and hence the functional distribution of the income from that factor should change. Other applications include circumstances where there is a labour-leisure trade-off at the level of the utility functions of households and in dynamic applications where patterns of capital accumulation, and hence ownership, vary across institutions. More generally the formulation must encompass the implications of household migration and factor market mobility (see below for details).

This model however extends the treatment of the factor markets by segmenting factor accounts. The ‘standard/classical’ treatment of factors markets involves the presumption that factors are solely differentiated by reference to the factor category, e.g., ‘skilled’ labour, but the underlying data often indicate that factors prices/wage rates differ markedly by activity. Consequently, if labour within a category are assumed to be perfectly mobile across all activities this requires the implicit assumption that all differences in wage rates by activities are due to activity specific determinants. But, it is likely that each labour category embraces a range of heterogenous types of labour and that some of the differences in wage rates are attributable to differences in labour. The model therefore allows labour within a category employed by an activity, or group of activities, to transition with imperfect substitutability into labour of the same a category working in another set of activities, either adopting the productivity in the activities they have moved to, or taking the productivity they had in their previous activity of some intermediate level of productivity (this is a development of McDonald and Thierfelder (2008) and Flaig *et al*., 2013).

Furthermore, there are no substantive *a priori* reasons to argue that labour cannot transition between categories, e.g., agricultural workers transitioning into construction workers.[10] Thus this model contains a generalisation of the method in Flaig, *et al*., 2013, that allows labour to transition between all segments defined by the types of labour and the activities or group of activities. Not all transition pathways are likely to be open, so the user defines those pathways that are open; nor is the ease of transition likely to be equally easy for all pathways so the user needs to specific pathway specific elasticities.

Similarly, there are no *a priori* reasons to believe that responses to economic policy changes and/or shocks will not give 'individual' households incentives to transition from one RHG to another RHG. The possibilities for households to transition and the ease with which they can transition depend, in part, on the classification scheme selected. For instance, a 'standard' classification scheme may distinguish between rural and urban RHGs, in which case one or more rural-urban migration pathways may be available and conventional economic arguments about incentives to migrate (after Harris and Todaro) can be invoked. Whereas a classification scheme based on educational qualifications of the 'head of household' is likely to more restrictive, at least in the short run, since it takes time to acquire 'new' qualifications, and for some time periods completely close off some pathways, e.g., from a 'head of household' who is illiterate to one who has a tertiary qualification.

Thus, as with the factor mobility functions, the user defines those pathways that are open; nor is the ease of transition likely to be equally easy for all pathways so the user needs to specific pathway specific elasticities. The degree (elasticity) of migration of each 'individual' household between different RHGs can be varied between zero (perfectly

[10] It is a common practice to find labour categories in SAMs being defined by reference to occupational categories, often those defined by the ILO, which is a good method for describing the current allocation of labour but limiting with respect to analysing how labour may reallocate following a policy change/shock. If this is the case then certain labour categories are likely to be dominant in some activities and rare in others, e.g., shop workers common in retail activities but rare in agriculture, which provides little evidence about the ability of labour to move to other activities.

immobile) and near infinity (nearly perfectly mobile) depending on model settings selected by the user and changes in relative household incomes.

The interplay of factor mobility and RHG migration, together with employment changes and the labour-leisure trade-offs, changes the patterns of factor ownership by each RHG and hence changes the functional distribution of income. Consequently, if any factor mobility pathway or household migration pathway is render open by the user, the functional distribution of income must be endogenously determined, i.e., controlled by variables, if the model is to make economic sense.

The other behavioural relationships in the model are generally linear. A few features do however justify mention. First, all the tax rates are declared as variables with various adjustment and/or scaling factors that are declared as variables or parameters according to how the user wishes to vary tax rates. If a fiscal policy constraint is imposed then one or more of the sets of tax rates can be allowed to vary equiproportionately and/or additiviely to define a new vector of tax rates that is consistent with the fiscal constraint. Relative tax rates can also be adjusted by the settings chosen by the user. Similar adjustment and/or scaling factors are available for a number of key parameters, e.g., household and enterprise savings rates and inter-institutional transfers. Second, technology changes can be introduced through changes in the activity specific efficiency variables – adjustment and/or scaling factors are also available for the efficiency parameters. Third, the proportions of current expenditure on commodities defined to constitute subsistence consumption can be varied. Fourth, although a substantial proportion of the sub matrices relating to transfers, especially with the rest of the world, contain zero entries, the model allows changes in such transfers, e.g., aid transfers to the government from the rest of the world may be defined equal to zero in the database but they can be made positive, or even negative, for model simulations. And fifth, the model is set up with a range of flexible macroeconomic closure rules and market clearing conditions. While the base model has a standard neoclassical model closure, e.g., full employment, savings driven investment and a floating exchange rate, these closure conditions can all be readily altered.

Transaction Relationships

The transactions relationships are laid out in Table 3, which is in two parts. The prices of domestically consumed (composite) commodities are defined as PQD_c, and they are the same irrespective of which agent purchases the commodity, except for final demand by households on which VAT can be levied. The quantities of commodities demanded domestically are divided between intermediate demand, $QINTD_c$, and final demand, with final demand further subdivided between demands by households, QCD_c (natural commodities) and $QCD2_{cag}$ (aggregate commodities from the nested utility functions), enterprises, $QENTD_c$, government, QGD_c, investment, $QINVD_c$, and stock changes, $dstocconst_c$. The value of total domestic demand, at purchaser prices, is therefore $PQD_c * QQ_c$. Consequently the decision to represent export demand, QE_c, as an entry in the commodity row is slightly misleading, since the domestic prices of exported commodities, $PE_c = PWE_c * ER$, do not accord with the law of one price. The representation is a space saving device that removes the need to include separate rows and columns for domestic and exported commodities.[11] The price wedges between domestic and exported commodities are represented by export duties, TE_c, that are entered into the commodity columns. Commodity supplies come from domestic producers who receive the common prices, PXC_c, for outputs irrespective of which activity produces the commodity, with the total domestic production of commodities being denoted as QXC_c. Commodity imports, QM_c, are valued carriage insurance and freight (cif) paid, such that the domestic price of imports, PM_c, is defined as the world price, PWM_c, times the exchange rate, ER, plus an *ad valorem* adjustment for import duties, TM_c. All domestically consumed commodities are subject to a variety of product taxes, sales taxes, TS_c, excise taxes, TEC_c. and value added taxes, TV_c. Other taxes can be readily added.

Domestic production activities receive average prices for their output, PX_a, that are determined by the commodity composition of their outputs. Since activities produce multiple outputs their outputs can be represented as an index, QX_a, formed from the commodity composition of their outputs. In addition to intermediate inputs, activities also purchase primary inputs, $FD_{f,a}$, for which they pay average prices, WF_f. To create greater flexibility the model allows the price of each factor to vary according to the activity that employs the factor. Finally each activity pays production taxes, the rates, TX_a, for which are proportionate to the value of activity outputs.

The model allows for the domestic use of both domestic and foreign owned factors of production, and for payments by foreign activities for the use of domestically owned factors.

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11 In this model the allocation by domestic producers of commodities between domestic and export markets is made on the supply side; implicitly there are two supply matrices – supplies to the domestic market and supplies to the export market.

Factor incomes therefore accrue from payments by domestic activities and foreign activities, $factwor_f$, where payments by foreign activities are assumed exogenously determined and are denominated in foreign currencies. After allowing for depreciation, $deprec_f$, and the payment of factor taxes, TF_f, the residual factor incomes, $YFDIST_f$, are divided between domestic institutions (households, enterprises and government) and the rest of the world in fixed proportions.

Households receive incomes from factor rentals and/or sales ($INSVASH_{h,f}$), inter household transfers, $hohoconst_{h,h}$, transfers from enterprises, $hoentconst_h$, and government, $hogovconst_h$, and remittances from the rest of the world, $howor_h$, where remittances are defined in terms of the foreign currency. Household expenditures consist of payments of direct/income taxes, TY_h, after which savings are deducted, where the savings rates, SHH_h, are fixed exogenously in the base configuration of the model. The residual household income is then divided between inter household transfers and consumption expenditures, with the pattern of consumption expenditures determined by the household utility functions.

Table 3 Transactions Relationships for the Standard Model

	Commodities	Activities	Factors	Households
Commodities	0	$\left(PQD_c * QINTD_c\right)$	0	$\left(PQD_{cles} * \left(1+TV_c\right) * QCD_{cles}\right)$ $\left(PQCD_{cag} * QCD2_{cag}\right)$
Activities	$\left(PXC_c * QXC_c\right)$ $\left(PX_a * QX_a\right)$	0	0	0
Factors	0	$\left(WF_f * FD_{f,a}\right)$	0	0
Households	0	0	$\sum_f INSVASH_{h,f}$	$\left(\sum_{hh} hohoconst_{hh,h}\right)$
Enterprises	0	0	$\left(\sum_f INSVASH_{e,f}\right)$	0
Government	$\left(TM_c * PWM_c * QM_c * ER\right)$ $\left(TE_c * PWE_c * QE_c * ER\right)$ $\left(TV_c * PQD_c * QCD_c\right)$ $\left(TS_c * PQS_c * QQ_c\right)$ $\left(TEC_c * PQS_c * QQ_c\right)$	$\left(TX_a * PX_a * QX_a\right)$	$\left(\sum_f INSVASH_{gt,f}\right)$ $\left(TF_f * YFDISP_f\right)$	$\left(TY_h * YH_h\right)$
Capital	0	0	$\sum_f deprec_f$	$\left(SSH_h * YH_h\right)$
Rest of World	$\left(PWM_c * QM_c * ER\right)$	0	$\left(\sum_f INSVASH_{w,f}\right)$	0
Total	$\left(PQD_c * QQ_c\right)$	$\left(PX_a * QX_a\right)$	YF_f	YH_h

Table 3 (cont) Transactions Relationships for the Standard Model

	Enterprises	Government	Capital	RoW	Total
Commodities	$(PQD_c * QENTD_c)$	$(PQD_c * QGD_c)$	$(PQD_c * QINVD_c)$ $(PQD_c * dstocconst_c)$	$(PWE_c * QE_c * ER)$	$(PQD_c * QQ_c)$
Activities	0	0	0	0	$(PX_a * QX_a)$
Factors	0	0	0	$(factwor_f * ER)$	YF_f
Households	$hoentconst_h$	$(hogovconst_h * HGADJ)$	0	$(howor_h * ER)$	YH_h
Enterprises	0	$(entgovconst * EGADJ)$	0	$(entwor * ER)$	$EENT$
Government	$(TYE * YE)$	0	0	$(govwor * ER)$	EG
Capital	$(YE - EENT)$	$(YG - EG)$	0	$(CAPWOR * ER)$	$TOTSAV$
Rest of World	0	0	0	0	Total 'Expenditure' Abroad
Total	YE	YG	$INVEST$	Total 'Income' from Abroad	

The enterprise account receives income from factor sales (*INSVASH*), primarily in the form of retained profits,[12] transfers from government, *entgovconst*, and foreign currency denominated transfers from the rest of the world, *entwor*. Expenditures then consist of the payment of direct/income taxes, *TYE*, consumption, which is assumed fixed in real terms,[13] and savings, which are defined as a residual, i.e., the difference between income, *YE*, and committed expenditure, *EENT*. There is an analogous treatment of government savings, i.e., the internal balance, which is defined as the difference (residual) between government income, *YG*, and committed government expenditure, *EG*. In the absence of a clearly definable set of behavioural relationships for the determination of government consumption expenditure, the quantities of commodities consumed by the government are fixed in real terms, and hence government consumption expenditure will vary with commodity prices.[14] Transfers by the government to other domestic institutions are fixed in nominal terms, although there is a facility to allow them to vary, e.g., with consumer prices. On the other hand government incomes can vary widely. Incomes accrue from the various tax instruments (import and export duties, sales, production and factor taxes, and direct taxes), that can all vary due to changes in the values of production, trade and consumption, and from factors (*INSVASH*). The government also receives foreign currency denominated transfers from the rest of the world, *govwor*, e.g., aid transfers.

Domestic investment demand consists of fixed capital formation, $QINVD_c$, and stock changes, $dstocconst_c$. The comparative static nature of the model and the absence of a capital composition matrix underpin the assumption that the commodity composition of fixed capital formation is fixed, while a lack of information means that stock changes are assumed invariant. However the value of fixed capital formation will vary with commodity prices while the volume of fixed capital formation can vary both as a consequence of the volume of savings changing or changes in exogenously determined parameters. In the base version of the model domestic savings are made up of savings by households, enterprises, the government (internal balance) and foreign savings, i.e., the balance on the capital account or external

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12 Hence the model contains the implicit presumption that the proportions of profits retained by incorporated enterprises are constant.

13 Hence consumption expenditure is defined as the fixed volume of consumption, $QENTD_c$, times the variable prices. It requires only a simple adjustment to the closure rules to fix consumption expenditures. Without a utility function, or equivalent, for enterprises it is not possible to define the quantities consumed as the result of an optimisation problem.

14 The closure rules allow for the fixing of government consumption expenditure rather than real consumption.

balance, *CAPWOR*. The various closure rules available within the model allow for different assumptions about the determination of domestic savings, e.g., flexible versus fixed savings rates for households, and value of 'foreign' savings, e.g., a flexible or fixed exchange rate.

Incomes to the rest of the world account, i.e., expenditures by the domestic economy in the rest of the world, consist of the values of imported commodities and factor services. On the other hand expenditures by the rest of the world account, i.e., incomes to the domestic economy from the rest of the world, consist of the values of exported commodities and NET transfers by institutional accounts. All these transactions are subject to transformation by the exchange rate. In the base model the balance on the capital account is fixed at some target value, denominated in foreign currency terms, e.g., at a level deemed equal and opposite to a sustainable deficit on the current account, and the exchange rate is variable. This assumption can be reversed, where appropriate, in the model closure.

Core Price and Quantity Relationships

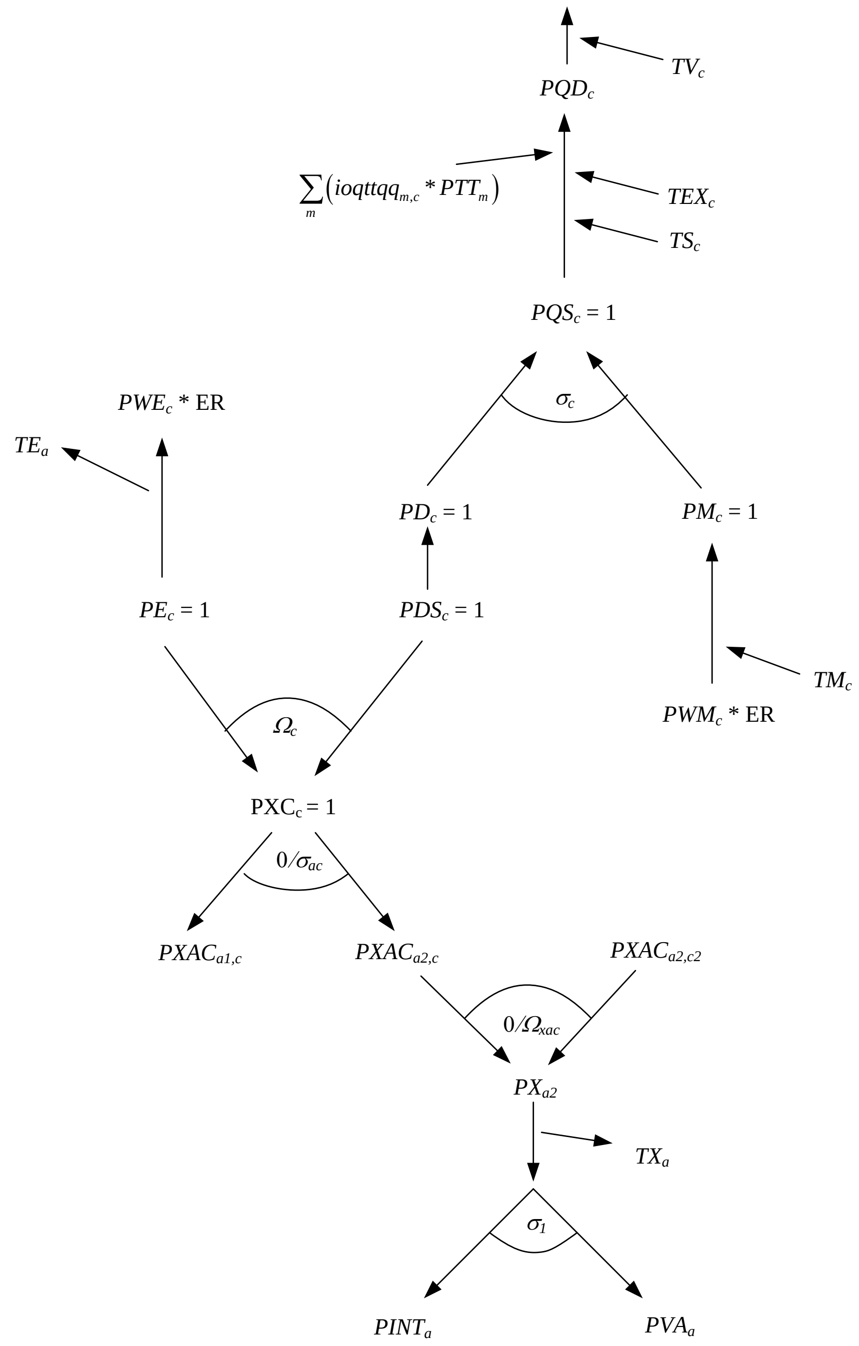
Figures 1 and 2 provide further detail on the interrelationships between the prices and quantities for commodities and activities. The supply prices of the composite commodities (PQS_c) are defined as the weighted averages of the domestically produced commodities that are consumed domestically (PD_c) and the domestic prices of imported commodities (PM_c), which are defined as the products of the world prices of commodities (PWM_c) and the exchange rate (ER) uplifted by *ad valorem* import duties (TM_c). These weights are updated in the model through first order conditions for optima. The average prices exclude sales taxes, and hence must be uplifted by (*ad valorem*) sales and excise taxes (TS_c , TEX_c), and possibly other tax instruments, and by trade and transport margins ($ioqttqq_{m,a} * PTT_m$) to reflect the composite consumer price (PQD_c).[15] The producer prices of commodities (PXC_c) are similarly defined as the weighted averages of the prices received for domestically produced commodities sold on domestic and export (PE_c) markets. These weights are updated in the model through first order conditions for optima. The prices received on the export market are defined as the products of the world price of exports (PWE_c) and the exchange rate (ER) less any exports duties due, which are defined by *ad valorem* export duty rates (TE_c).

The average price per unit of output received by an activity (PX_a) is defined as the weighted average of the domestic producer prices ($PXAC_{a,c}$), where the weights are constant

[15] For simplicity only one tax on domestic commodity sales is included in this figure.

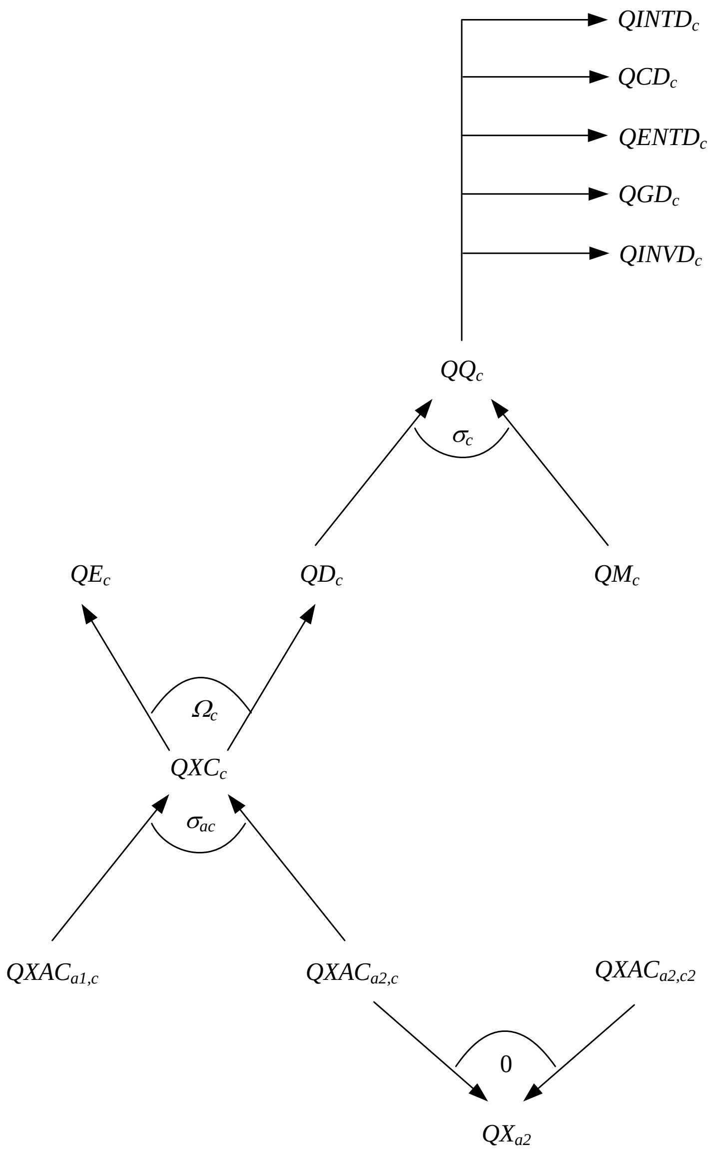
or variables according to the model configuration. After paying indirect/production/output taxes (TX_a), this is divided between payments to aggregate value added (PVA_a), i.e., the amount available to pay primary inputs, and aggregate intermediate inputs ($PINT_a$). Total payments for intermediate inputs per unit of aggregate intermediate input are defined as the weighted sums of the prices of the inputs (PQD_c).

Figure 1 Price Relationships for the STAGE Model



Total demands for the composite commodities, QQ_c, consist of demands for intermediate inputs, $QINTD_c$, consumption by households, QCD_c, enterprises, $QENTD_c$, and government, QGD_c, gross fixed capital formation, $QINVD_c$, and stock changes, $dstocconst_c$. Supplies from domestic producers, QD_c, plus imports, QM_c, meet these demands; equilibrium conditions ensure that the total supplies and demands for all composite commodities equate. Commodities are delivered to both the domestic and export, QE_c, markets subject to equilibrium conditions that require all domestic commodity production, QXC_c, to be either domestically consumed or exported.

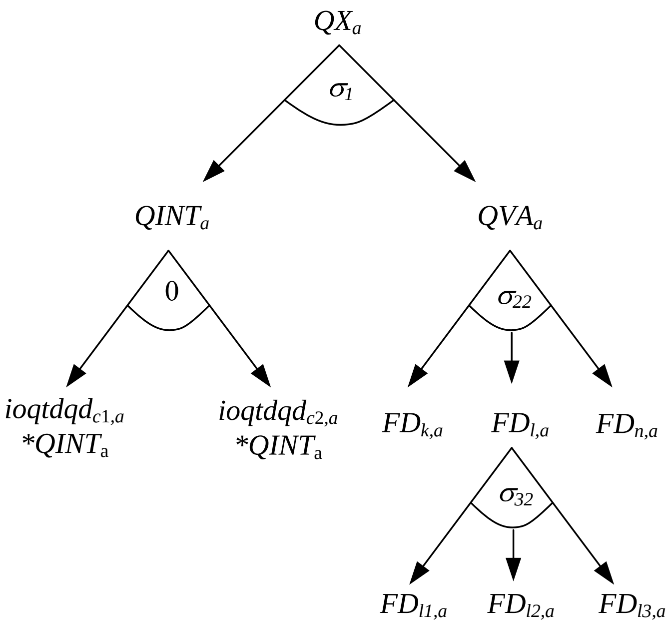
Figure 3 Quantity Relationships for the STAGE



Production Relationships

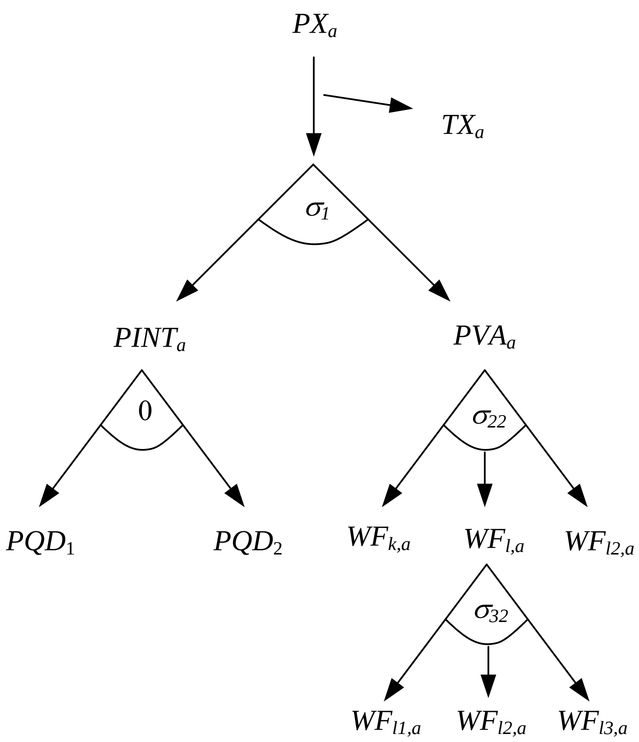
The presence of multiple product activities means that domestically produced commodities can come from multiple activities, i.e., the total production of a commodity is defined as the sum of the amount of that commodity produced by each activity. Hence the domestic production of a commodity (*QXC*) is a CES aggregate of the quantities of that commodity produced by a number of different activities (*QXAC*), which are produced by each activity in activity specific fixed proportions, i.e., the output of *QXAC* is a Leontief (fixed proportions) aggregate of the output of each activity (*QX*).

Figure 4 Production Relationships for the STAGE Model: Quantities



Production relationships by activities are defined by a series of nested Constant Elasticity of Substitution (CES) production functions. In the base version there is a two level production nest, which, in quantity terms, is illustrated in Figure 4. For illustration purposes only, two intermediate inputs and five primary inputs ($FD_{k,a}$, $FD_{l1,a}$, $FD_{l2,a}$, $FD_{l3,a}$ and $FD_{n,a}$) together with one aggregate primary input ($FD_{l,a}$) are identified. Activity output is a CES aggregate of the quantities of aggregate intermediate inputs (*QINT*) and value added (*QVA*), while aggregate intermediate inputs are a Leontief aggregate of the (individual) intermediate inputs and aggregate value added is a CES aggregate of the quantities of two primary and one aggregate inputs demanded by each activity (*FD*). The aggregate primary input is then a CES aggregate of the different primary factors at the third level. The allocation of the finite supplies of factors (*FSI*) between competing activities depends upon relative factor prices via first order conditions for optima.

Figure 5 Production Relationships for the STAGE Model: Prices



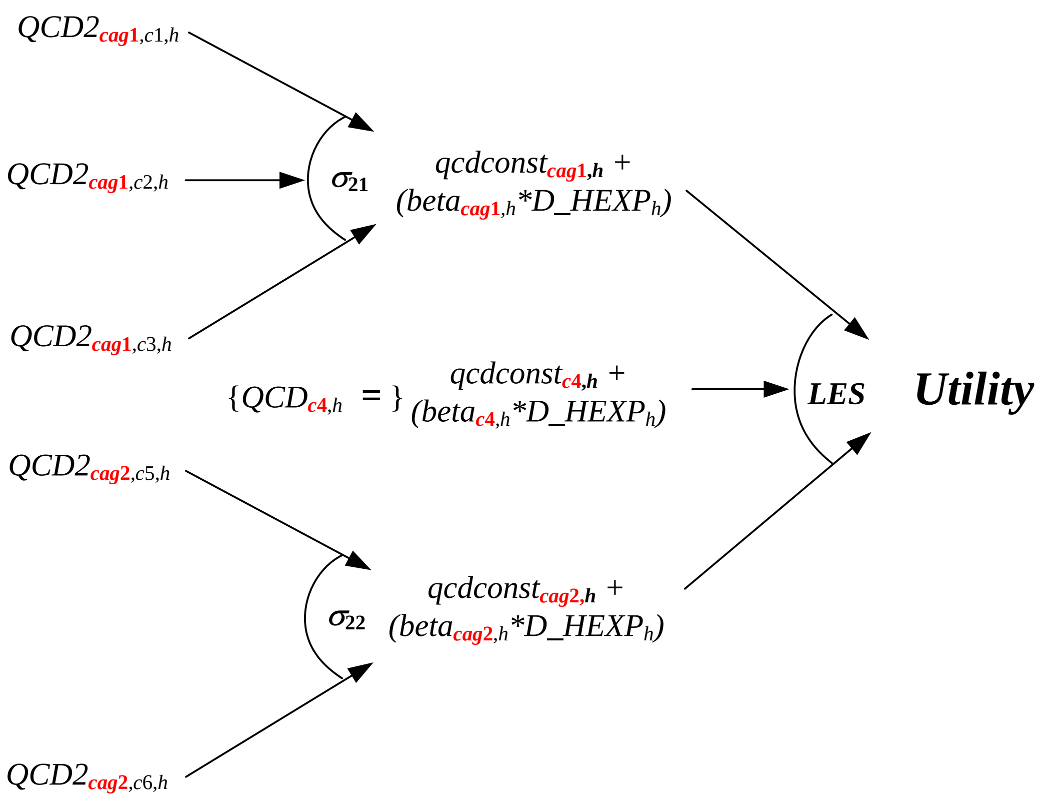
The price relations for the production system are illustrated in Figure 5. Note how the prices paid for intermediate inputs (*PQD*) are the same as paid for final demands, i.e., a ‘law’ of one price relationship holds across all domestic demand. Note also that factor prices are factor and activity specific ($WF_{f,a}$).

<u>Household Utility Relationships</u>

The nested (CES-LES) utility functions have a linear expenditure system (LES) defined over a mix of natural and aggregate commodities. This is illustrated in quantity terms in Figure 6 where the subscript ‘*cag#*’ indicates an aggregate commodity and the subscript ‘*c#*’ a natural commodity. The underlying logic is that each household demands subsistence quantities of certain aggregate commodities, e.g., food, energy, etc., but not necessarily of all natural commodities, e.g., meat, gas, etc. Thus the LES utility functions for each household are defined over a mix of aggregate and natural commodities demands for which there are subsistence quantities (*qcdconst*) and marginal budget shares (*beta*) of discretionary household consumption expenditures (*D_HEXP*).

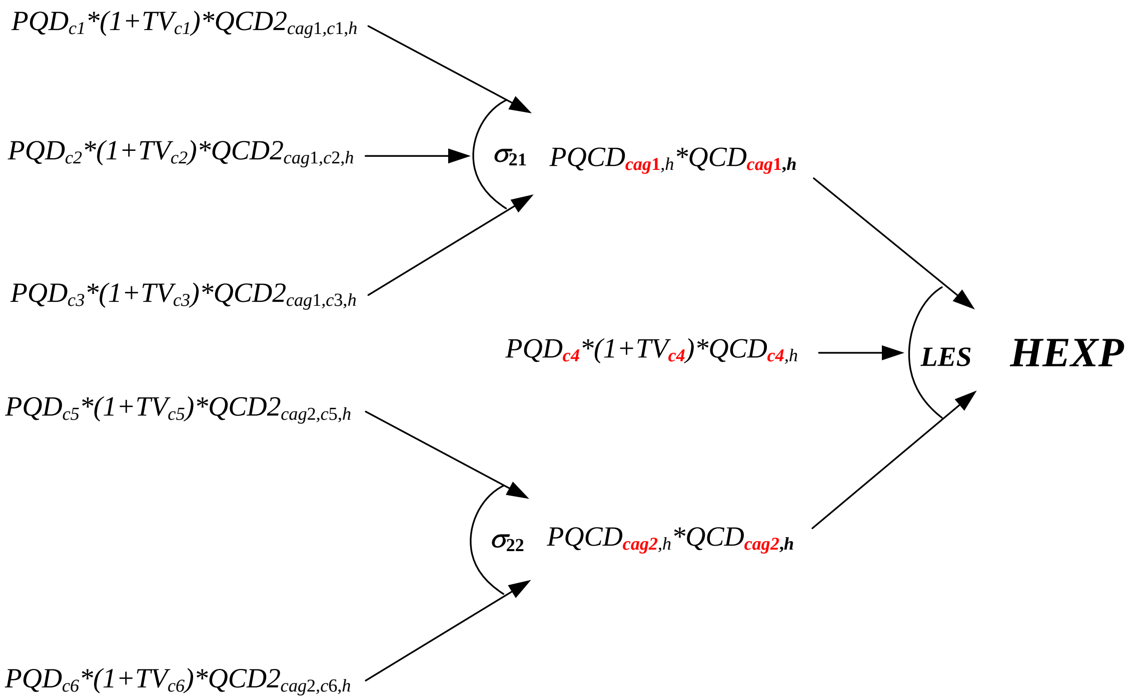
The aggregate commodities are CES aggregates of various natural commodities that are demanded to generate the aggregate. Since each household, *h*, has different preferences, as disclosed by the data, the quantities of each commodity, *c*, used to generate an aggregate, *cag*, the demand for each commodity (*QCD*2) has three arguments. As illustrated in Figure 6 the system is general in the sense that any number of commodities can be used to generate each aggregate and there can be any mix of aggregates and natural commodities in the LES.

Figure 6 Utility Functions in Quantities



The Law of One Price (LOOP) must however be retained. Thus despite the demand for commodities by each household depending on *c* and *h* the prices paid are only determined by the commodity *c*. However, since the mix of commodities in each aggregate commodity varies by household because the quantities of each natural commodity, the weights, are different for each household. Consequently the aggregate prices (*PQCD*) are indexed on both the aggregate commodity, *cag*, and the household, *h*. This is illustrated in Figure 7 where the components of the transaction values are identified.

Figure 7 Utility Functions in Transaction Values



Two points deserve emphasis. The prices for aggregate commodities entering into the LES function cannot, by definition, be charged VAT (*TV*) since they have no real equivalent.[16] But the value of the aggregate commodities must be defined so that they include any VAT paid on the natural commodities that make up the aggregate commodities. And second, the requirement for elasticity estimates is both increased and made empirically more difficult. The increase in empirical difficulty is that substitution elasticities are required for the components of the aggregate commodities; the income elasticities of demand are for aggregates, which is actually the situation encountered with standard implementations of LES functions.

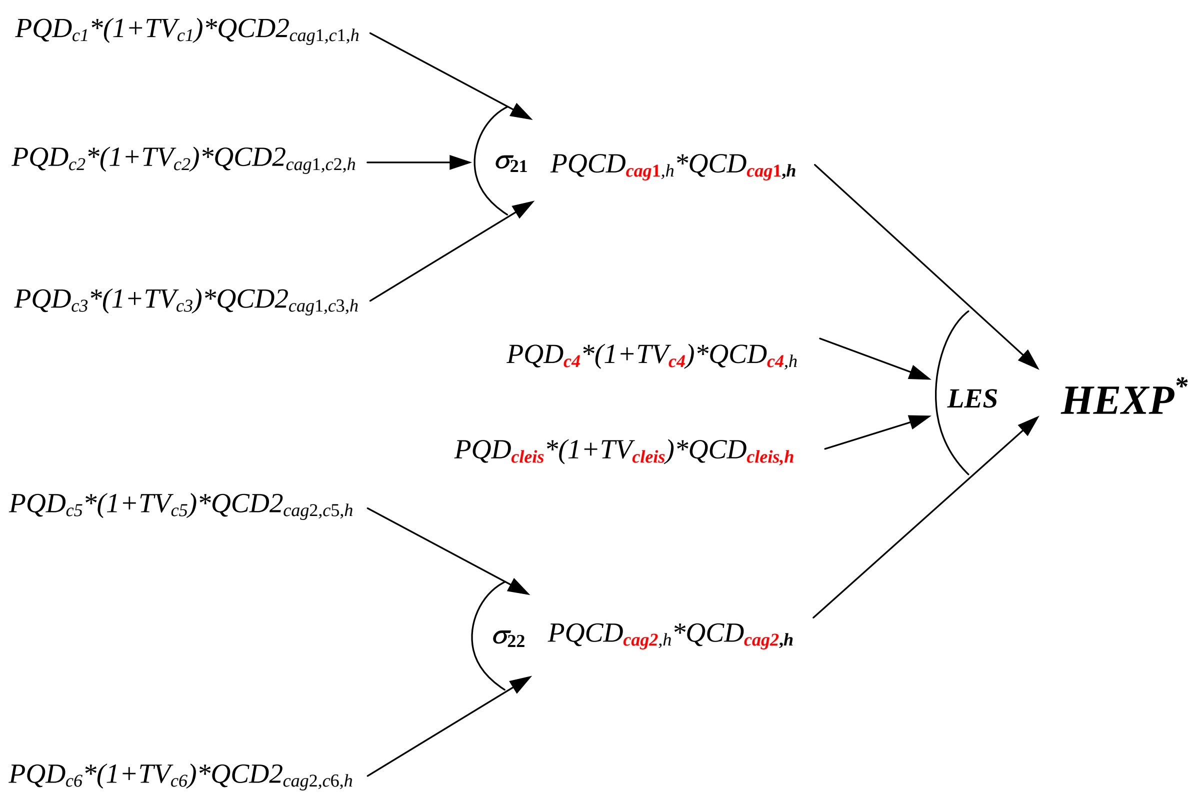
<u>Labour-Leisure Trade Off</u>

Leisure is introduced into the model as 'special' commodities that are RHG specific and can only be produced using labour supplied by the RHG that consumes that type of leisure. Since the compositions and quantities of the labour inputs used to produce leisure are specific to

16 The model code appears to include VAT on aggregate commodities in the LES equations in the nest. This is because of how the equations are written; elsewhere in the code the VAT rates on aggregate commodities are fixed at zero. However it is important to guard against creating a VAT rate on aggregates in policy simulations; to do so the user would need to override controls included to avoid such a possibility.

each RHG, each type of leisure has a unique input mix and hence cost of production and price. Thus the production system is extended to include one leisure activity for each RHG, with each leisure activity producing one RHG specific leisure commodity that can only be produced by that activity. The RHG specific leisure commodities then enter in to model utility functions at the LES level, which simply requires an extension to the nested utility functions to include leisure (see Figure 8).

Figure 8 Utility Functions with Leisure



This formulation implies that leisure accounts for a positive marginal budget share (*beta*) so that as RHG consumption expenditures increase so does the demand for leisure; the income effect. The substitution effect depends on the prices of the 'commodities' in the LES nest. Since the prices of the leisure commodities depends solely on the wage rates and labour input shares the behavioural relationships underpinning the substitution effects are the marginal costs of producing leisure by the leisure activities that are linked directly to individual RHG.

Note however that in household consumption variable in Figure 8 (*HEXP**) is different to the household consumption variable in Figure 7 (*HEXP*). This highlights the requirement that the household consumption variable with labour-leisure trade-offs must include the value of leisure. This raises the empirical problem of how to value leisure.

The opportunity cost of labour used in the production of leisure is the marginal wage income foregone; hence the transaction values in the SAM database for labour used in leisure activities are the wage rates for each labour type times the quantities of labour used to produce leisure. Since in this context leisure time is time foregone from the labour market, within the production boundary, by members of the RHGs its valuation avoids the complication associated with defining the production boundary. Specifically leisure time can only be provided by those persons that enjoy the leisure and the opportunity cost of the time and its market price are identical, and therefore leisure can be given an unambiguous price and hence valuation. Thus the problem of valuing leisure reduces to deriving estimates of the time 'devoted' to leisure EXCLUDING time taken up providing services outwith the SNA production boundary.[17]

Endogenous Functional Distribution of Income

Any formulation of a CGE model that allows endogenous changes in factor supply requires some method, implicit or explicit, for assigning the changes in factors supply by each institution to the existing structure/pattern of factor supply. In most known CGE models where the code can be verified, e.g., IFPRI, STAGE 1 and PEP, the allocation is implicit: the structure/pattern of factor supply by institution is fixed, i.e., model parameters, which amounts to an assumption that changes in factor supplies are drawn from institutions proportionate to their supplies in the base period. It is arguable that this assumption, a fixed and exogenously determined functional distribution of income, is generally inappropriate.

A standard assumption is a perfectly elastic supply of labour at the existing wage rate/marginal cost. The assumption that the functional distribution is fixed requires that any labour added/subtracted from the market is drawn/withdrawn equiproportionately from all institutions. Consider the simple case of 2 households, one rich and one poor, both supplying skilled and unskilled labour. It is arguably likely that the rich household will have more

---

[17] This should not be interpreted as an endorsement of the SNA's definition of the production boundary. There is a long literature on the issue of the valuation of services provided outside of the SNA's production boundary. In essence the contrasting arguments can be reduced to whether labour used to produce such services should be valued on an opportunity cost basis, i.e., child care provided by high earners should have a higher valuation that that provided by low earners, or a market prices, i.e., child care should be valued at the cost of hiring nannies.

skilled labour relative to the poor household, and hence that there is the possibility that any increase in supply of unskilled labour may be biased towards the poor household. However it may arise, ANY changes in labour supplies are likely to generate some changes in the pattern of factor supply by institution, and that given any systematic behavioural determinants of changes in labour supply changes in the functional distribution of income is highly probable

This model introduces behavioural relationships that ensure the functional distribution of income changes as the patterns of factor supply by institution change.

Factor Mobility Relationships

The typical approach to factor supply and demand is that factors are rigidly segmented so that the total available supply of a factor is fixed, either at the current level of total demand, i.e., full employment, or at some level that is greater than the current level of total demand so as to allow for unemployment of that type of factor. This presumption of rigid segmentation is restrictive and as such does not allow for the possibility that labour can transition, to a greater or lesser extent, between the segments in response to changes in the factor market with or without job specific training. For instance tractor operators, in agriculture, may readily transition into JCB operators, in construction. Moreover the common option of classifying labour by levels of skill, e.g., skilled, semi-skilled and unskilled, involves the implicit assumption that all labour of a specific type receives the same average wage rate when within each type of labour there is likely to be a range of wage rates about the average. Thus it may be that lower paid skilled workers may be willing to take employment as semi-skilled workers if average wage rates for skilled workers decline relative to those of semi-skilled workers.

These differences in average wage rates are reflected in the fact that labour of the same type are typically paid at different wage rates according to the activity in which they are employed. This is transparent in any CGE model for which there are data for both the transactions values and quantities of labour employed by different activities and are reported as differences in the values of $WFDIST_{f,a}$. Where such data are available it is not uncommon to find that skilled agricultural workers are paid less than semi-skilled manufacturing workers. Such an observation implies, within the logic of CGE models and the labour classification scheme, that the labour market is not operating efficiently. Such an observation is typically, if at all, justified by some combination of assuming that there are non-rewarded preferences that explain the differences in average wages and/or that the differences are entirely due to activity specific attributes, i.e., the maintained assumption is that the labour classification scheme encompasses all differences in characteristics of the labour types.

The inclusion of the assumption of factor mobility across types of labour relaxes this restrictive assumption by assuming that types of factors can transition into other, specified, types in response to changes in the relative rates of return to factor. This achieved by segmenting the demand for specific types of factor according to the activity, or group of activities, and introducing factor mobility functions that allow a factor to move between designated segments with imperfect mobility. There also a need to ensure that no additional factors are created so as to ensure that the factor market clearing conditions are expressed in 'natural' physical quantity units; this achieved by a constraint equation that ensures that for each unit of a factor moved from one segment only one unit of a factor is created in the paired segment. This approach was first developed in the STAGE_LAB model (McDonald and Thierfelder, 2007), was refined in Dorothee Flaig's PhD (see Flaig, *et al*., 2014) and generalised in GLOBE_LAB (McDonald *et al*., 2015), which provides the basis of the formulation used in STAGE_DEV.

The endogeniety of the functional distribution is ensured by the fact that the functional distribution of income depends upon the shares of factors supplied by different institutions, which is a function of the supply of factors by institutions. Since both of these are endogenous variables the functional distribution of income is endogenous.

Institution Migration Block

The typical approach to modelling households, one of the groups that make up institutions, is to assume that representative household groups (RHG) are rigidly segmented and that each RHG receives a fixed share of factor incomes generated domestically and received from abroad. The first part of the assumption requires that households are not allowed to migrate, e.g., rural-urban migration is precluded, while the second part carries the implicit presumption that any changes in factor supplies by RHGs changes the supply of each factor by each RHG equiproportionately. Neither of these assumptions is necessarily an accurate reflection of reality and, at the same time, imposes restrictions that require households to NOT to change behaviour in response to economic signals. The STAGE_DEV model allows RHGs to relocate/migrate in response to changes in economic signals, e.g., changes in relative RHG incomes, and when they do so to transfer their factors from the RHG they leave to the RHG they join.

The inclusion of the assumption of household migration across types of household relaxes the restrictive assumption of rigid segmentation of RHGs. The behavioural assumption is that the incentives for RHG to migrate in the model are changes in the relative returns to different RHGs, i.e., only economic incentives are embodied within the behavioural assumption. Thus if RHGs in one segment experience a RELATIVE increase in income to the RHGs in all other segments there will be incentives for households to relocate/migrate to the RHG that receives an increase in RELATIVE income. There are two things about this relationship that are important to note: first it is assumed that all other influences on the location decisions of RHGs are unchanged and second that the user needs to define the pairs of RHGs between which households can migrate.

Given these changes in relative incomes and elasticities of migration, which are factor and institution pair specific, the quantities of households moving between RHG pairs for each institution can be determined and the supply of each factor by each institution (*FSI*) can be deduced. There is also a need to ensure that no additional households are created; this achieved by a constraint equation that ensures that for each household moved from one segment only one household is created in the paired segment.

The endogeniety of the functional distribution is ensured by the fact that the functional distribution of income depends upon the shares of factors supplied by different institutions, which is a function of the supply of factors by institutions. Since both of these are endogenous variables the functional distribution of income is endogenous.

Algebraic Statement of the Model

The model uses a series of sets, each of which is required to be declared and have members assigned. For the majority of the sets the declaration and assignment takes place simultaneously in a single block of code.[18] However, the assignment for a number of the sets, specifically those used to control the modeling of trade relationships is carried out

18 For practical purposes it is often easiest if this block of code is contained in a separate file that is then called up from within the *.gms file. This is how the process is implemented in the worked example.

dynamically by reference to the data used to calibrate the model. The following are the basic sets for this model

$$
\begin{aligned}
cc &= \{\text{natural and aggregate commodities}\} \\
c &= \{\text{natural commodities}\} \\
a &= \{\text{activities}\} \\
f &= \{\text{factors}\} \\
ins &= \{\text{domestic institutions}\} \\
insw &= \{\text{domestic institutions and rest of the world}\} \\
h &= \{\text{households}\} \\
g &= \{\text{government}\} \\
e &= \{\text{enterprises}\} \\
i &= \{\text{investment}\} \\
w &= \{\text{rest of the world}\}
\end{aligned}
$$

and for each set there is an alias declared that has the same membership as the corresponding basic set. The notation used involves the addition of a '*p*' suffix to the set label, e.g., the alias for *c* is *cp*.

However, for practical/programming purposes these basic sets are declared and assigned as subsets of a global set, *sac*,

$$sac = \{c, a, f, h, g, e, i, w, total\}.$$

All the dynamic sets relate to the modeling of the commodity and activity accounts and therefore are subsets of the sets *c* and *a*. The subsets are

$$
\begin{aligned}
ce(c) &= \{\text{export commodities}\} \\
cen(c) &= \{\text{non-export commodities}\} \\
ced(c) &= \{\text{export commodities with export demand functions}\} \\
cedn(c) &= \{\text{export commodities without export demand functions}\} \\
cm(c) &= \{\text{imported commodities}\} \\
cmn(c) &= \{\text{non-imported commodities}\} \\
cx(c) &= \{\text{commodities produced domestically}\} \\
cxn(c) &= \{\text{commodities NOT produced domestically AND imported}\} \\
cd(c) &= \{\text{commodities produced AND demanded domestically}\} \\
cdn(c) &= \{\text{commodities NOT produced AND demanded domestically}\}
\end{aligned}
$$

and members are assigned using the data used for calibration. Additionally there are some sets, referring to commodities and activities, which are used to control the behavioural equations implemented in specific cases. These are

$$
\begin{aligned}
cxac(c) &= \{\text{differentiated commodities produced domestically}\} \\
cxacn(c) &= \{\text{UNdifferentiated commodities produced domestically}\} \\
cles(cc) &= \{\text{natural and aggregate commodities in top level LES utility functions}\} \\
cces(c) &= \{\text{natural commodities in second level CES utility functions}\} \\
cag(cc) &= \{\text{aggregate commodities from second level CES utility functions}\} \\
aqx(a) &= \{\text{activities with CES aggregation at Level 1}\} \\
aqxn(a) &= \{\text{activities with Leontief aggregation at Level 1}\} \\
acet(a) &= \{\text{activities with CET aggregation of commodity outputs}\} \\
acetn(a) &= \{\text{activities with by-product - fixed proportion - aggregation of commodity outputs}\}
\end{aligned}
$$

and their memberships are set during the model calibration phase.

Finally a set is declared and assigned for a macro SAM that is used to check model calibration. This set and its members are

$$ss = \{commdty, activity, valuad, hholds, entp, govtn, kapital, world, totals\}.$$

Reserved Names

The model also uses a number of names that are reserved, in addition to those specified in the set statements detailed above. The majority of these reserved names are components of the government set; they are reserved to ease the modeling of tax instruments. The required members of the government set, with their descriptions, are

$$
g = \left\{ tf, \begin{array}{cc}
IMPTAX & \text{Import Taxes} \\
EXPTAX & \text{Export Taxes} \\
SALTAX & \text{Sales Taxes} \\
ECTAX & \text{Excise Taxes} \\
VATTAX & \text{VAT Taxes} \\
INDTAX & \text{Indirect Taxes} \\
FACTTAX & \text{Factor Taxes} \\
DIRTAX & \text{Direct Taxes} \\
GOVT & \text{Government}
\end{array} \right\}
$$

where *tf* is the set of factor use taxes, with one member of *tf* for each member of the set of factors, *f*.

The other reserved names are for the factor account and for the capital accounts. For simplicity the factor account relating to residual payments to factors has the reserved name of *GOS* (gross operating surplus); in many SAMs this account would include payments to the factors of production land and physical capital, payments labeled mixed income and payments for entrepreneurial services. Where the factor accounts are fully articulated *GOS* would refer to payments to the residual factor, typically physical capital and entrepreneurial services.

The capital account includes provision for multiple expenditure accounts relating to investment. All expenditures on stock changes are registered in the account *dstoc*, while all investment expenditures are registered to investment accounts *k***. All incomes to the capital account accrue to the *i_s* account and stock changes are funded by an expenditure levied on the *i_s* account to the *dstoc* account.

Conventions

The equations for the model are set out in eleven 'blocks'; which group the equations under the following headings 'trade', 'commodity price', 'numéraire', 'production', 'factor', 'household', 'enterprise', 'government', 'kapital', 'foreign institutions' and 'market clearing'. This grouping of equations is intended to ease the reading of the model rather than being a requirement of the model; it also reflects the modular structure that underlies the programme and which is designed to simplify model extensions/developments.

A series of conventions are adopted for the naming of variables and parameters. These conventions are not a requirement of the modeling language; rather they are designed to ease reading of the model.

- All VARIABLES are in upper case.
- The standard prefixes for variable names are: *P* for price variables, *Q* for quantity variables, *E* for expenditure variables, *Y* for income variables, and *V* for value variables
- All variables have a matching parameter that identifies the value of the variable in the base period. These parameters are in upper case and carry a '0' suffix, and are used to initialise variables.

- A series of variables are declared that allow for the equiproportionate adjustment of groups of parameters. These variables are named using the convention ***ADJ*, where ** is the parameter series they adjust.
- All parameters are in lower case, except those used to initialise variables.
- Names for parameters are derived using account abbreviations with the row account first and the column account second, e.g., *actcom*** is a parameter referring to the activity:commodity (supply or make) sub-matrix;
- Parameter names have a two or five character suffix which distinguishes their definition, e.g., ***sh* is a share parameter, ***av* is an average and ***const* is a constant parameter;
- The names for all parameters and variables are kept short.

Trade Block Equations

Trade relationships are modeled using the Armington assumption of imperfect substitutability between domestic and foreign commodities. The set of eleven equations are split across two sub-blocks – exports and imports - and provide a general structure that accommodates most eventualities found with single country CGE models. In particular these equations allow for traded and non-traded commodities while simultaneously accommodating commodities that are produced or not produced domestically and are consumed or not consumed domestically and allowing a relaxation of the small country assumption of price taking for exports.

Exports Block

The domestic price of exports (E1) is defined as the product of the world price of exports (*PWE*), the exchange rate (*ER*) and one minus the export tax rate[19]and are only implemented for members of the set *c* that are exported, i.e., for members of the subset *ce*. The cost of transporting commodities in the form of prices of per unit margin services are also included in determining PE_c. The world price of imports and exports are declared as variables to allow relaxation of the small country assumption, and are then fixed as appropriate in the model closure block.

The output transformation functions (E2), and the associated first-order conditions (E3), establish the optimum allocation of domestic commodity output (*QXC*) between domestic demand (*QD*) and exports (*QE*), by way of CET functions, with commodity specific share

[19] ALL tax rates are expressed as variables. How the tax rate variables are modeled is explained below.

parameters (γ), elasticity parameters (*rhot*) and shift/efficiency parameters (*at*). The first order conditions define the optimum ratios of exports to domestic demand in relation to the relative prices of exported (*PE*) and domestically supplied (*PD*) commodities. But (E2) is only defined for commodities that are both produced and demanded domestically (*cd*) **and** exported (*ce*). Thus, although this condition might be satisfied for the majority of commodities, it is also necessary to cover those cases where commodities are produced **and** demanded domestically but **not** exported, and those cases where commodities are produced domestically **and** exported but **not** demanded domestically.

Export Block Equations

$$PE_c = PWE_c * ER * (1 - TE_c) - \sum_m (ioqttqe_{m,c} * PTT_m) \qquad \forall ce \qquad \text{(E1)}$$

$$PE.FX_c = 0.0 \qquad \not\forall ce \qquad \text{(E1B)}$$

$$QXC_c = at_c * \left(\gamma_c * QE_c^{rhot_c} + (1 - \gamma_c) * QD_c^{rhot_c}\right)^{\frac{1}{rhot_c}} \qquad \forall ce \textbf{ AND } cd \qquad \text{(E2)}$$

$$QXC.FX_c = 0.0 \qquad \not\forall cx \qquad \text{(E2b)}$$

$$\frac{QE_c}{QD_c} = \left[\frac{PE_c}{PD_c} * \frac{(1 - \gamma_c)}{\gamma_c}\right]^{\frac{1}{(rhot_c - 1)}} \qquad \forall ce \textbf{ AND } cd\ . \qquad \text{(E3)}$$

$$QE.FX_c = 0.0 \qquad \not\forall ce \qquad \text{(E3b)}$$

$$QD.FX_c = 0.0 \qquad \not\forall cd \qquad \text{(E3c)}$$

$$QXC_c = QD_c + QE_c \qquad \forall (cen \textbf{ AND } cd) \textbf{ OR } (ce \textbf{ AND } cdn) \qquad \text{(E4)}$$

$$QE_c = econ_c * \left(\frac{PWE_c}{pwse_c}\right)^{-eta_c} \qquad \forall ced \qquad \text{(E5)}$$

If commodities are produced domestically but **not** exported, then domestic demand for domestically produced commodities (*QD*) is, by definition (E5), equal to domestic commodity production (*QXC*), where the sets *cen* (commodities not exported) and *cd* (commodities produced and demanded domestically) control implementation. On the other hand if commodities are produced domestically but **not** demanded by the domestic output, then domestic commodity production (*QXC*) is, by definition (E4), equal to commodity exports (*QE*), where the sets *ce* (commodities exported) and *cdn* (commodities produced but not demanded domestically) control implementation.

The equations E1 to E4 are sufficient for a general model of export relationships when combined with the small country assumption of price taking on all export markets. However, it may be appropriate to relax this assumption in some instances, most typically in cases where a country is a major supplier of a commodity to the world market, in which case it may be reasonable to expect that as exports of that commodity increase so the export price (*PE*) of that commodity might be expected to decline, i.e., the country faces a downward sloping export demand curve. The inclusion of export demand equations (E5) accommodates this feature, where export demands are defined by constant elasticity export demand functions, with constants (*econ*), elasticities of demand (*eta*) and prices for substitutes on the world market (*pwse*).

Imports Block

The domestic price of competitive imports (M1) is the product of the world price of imports (*PWM*), the exchange rate (*ER*) and one plus the import tariff rate (TM_c). These equations are only implemented for members of the set *c* that are imported, i.e., for members of the subset *cm*.

The domestic supply equations are modeled using Constant Elasticity of Substitution (CES) functions and associated first order conditions to determine the optimum combination of supplies from domestic and foreign (import) producers. The domestic supplies of the composite commodities (*QQ*) are defined as CES aggregates (M2) of domestic production supplied to the domestic market (*QD*) and imports (*QM*), where aggregation is controlled by the share parameters (δ), the elasticity of substitution parameters (*rhoc*) and the shift/efficiency parameters (*ac*). The first order conditions (M3) define the optimum ratios of imports to domestic demand in relation to the relative prices of imported (*PM*) and domestically supplied (*PD*) commodities. But (M2) is only defined for commodities that are both produced domestically (*cx*) **and** imported (*cm*). Although this condition might be satisfied for the majority of commodities, it is also necessary to cover those cases where commodities are produced but **not** imported, and those cases where commodities are **not** produced domestically **and** are imported.

Import Block Equations

$$PM_c = \left(PWM_c * \left(1 + TM_c\right)\right) * ER \qquad \forall cm. \tag{M1}$$

$$PM.FX_c = 0.0 \qquad \not\forall cm. \tag{M1b}$$

$$QQ_c = ac_c \left(\delta_c QM_c^{-rhoc_c} + \left(1-\delta_c\right) QD_c^{-rhoc_c}\right)^{-\frac{1}{rhoc_c}} \qquad \forall cm \textbf{ AND } cx \tag{M2}$$

$$\frac{QM_c}{QD_c} = \left[\frac{PD_c}{PM_c} * \frac{\delta_c}{\left(1-\delta_c\right)}\right]^{\frac{1}{\left(1+rhoc_c\right)}} \qquad \forall cm \textbf{ AND } cx. \tag{M3}$$

$$QM.FX_c = 0.0 \qquad \not\forall cm. \tag{M3b}$$

$$PD.FX_c = 0.0 \qquad \not\forall cd. \tag{M3c}$$

$$QQ_c = QD_c + QM_c \qquad \forall \left(cmn \textbf{ AND } cx\right) \textbf{ OR } \left(cm \textbf{ AND } cxn\right) \tag{M4}$$

If commodities are produced domestically but **not** imported, then domestic supply of domestically produced commodities (*QD*) is, by definition (M4), equal to domestic commodity demand (*QQ*), where the sets *cmn* (commodities not imported) and *cx* (commodities produced domestically) control implementation. On the other hand if commodities are **not** produced domestically but are demanded on the domestic market, then commodity supply (*QQ*) is, by definition (M4), equal to commodity imports (*QM*), where the sets *cm* (commodities imported) and *cxn* (commodities not produced domestically) control implementation.

Trade and Transport Margins Block

Trade and transport margins – margin services - record the costs of transferring commodities from their source (factory gate and port of entry) to consumer (domestic or foreign). At their sources commodities are valued at basic prices while at the point of consumption they are valued at purchaser prices, i.e., inclusive of indirect taxes and trade and transport margins.

The key assumption is that trade and transport margins are represented by the quantity of trade and transport services required to deliver a unit of the commodity to the consumer (*ioqttqq* and *ioqttqe* – for supplies to the domestic and foreign consumers respectively). Thus the quantity of trade and transport services required by the economy (*QTT*) is defined by the quantity of commodities demand times the quantity of margin services per unit of delivered commodity (M2).

The quantities of the commodities required (*QTTD*) to produce a unit of margins services are defined by Leontief technologies where the input coefficient ($ioqtdtt_{c,m}$) define the quantities of commodity *c* required to produce a unit of the margin services *m* (M3). Given the Leontief technologies the unit cost of the margin services (*PTT*) is a simple weighted average of the costs of the commodities used in its production (M1).

Trade and Transport Margins Block Equations

$$PTT_m = \sum_c ioqtddtt_{c,m} * PQD_c \ . \qquad \text{(M1)}$$

$$QTT_m = \sum_c \left(ioqttqq_{m,c} * QQ_c \right) + \sum_c \left(ioqttqe_{m,c} * QE_c \right). \qquad \text{(M2)}$$

$$QTTD_c = \sum_m ioqtddtt_{c,m} * QTT_m \ . \qquad \text{(M3)}$$

Commodity Price Block

The supply prices for commodities (P1) are defined as the volume share weighted sums of expenditure on domestically produced (*QD*) and imported (*QM*) commodities. These conditions derive from the first order conditions for the quantity equations for the composite commodities (*QQ*) above.[20] This equation is implemented for all commodities that are imported (*cm*) and for all commodities that are produced and consumed domestically (*cd*). Similarly, domestically produced commodities (*QXC*) are supplied to either or both the domestic and foreign markets (exported). The supply prices of domestically produced commodities (*PXC*) are defined as the volume share weighted sums of expenditure on domestically produced and exported (*QE*) commodities (P2). These conditions derive from the first order conditions for the quantity equations for the composite commodities (*QXC*) below.[21] This equation is implemented for all commodities that are produced domestically (*cx*), with a control to only include terms for exported commodities when there are exports (*ce*).

Commodity Price Block Equations

$$PQD_c = PQS_c * \left(1 + TS_c + TEX_c\right) + TQS_c + \sum_m \left(ioqttqq_{m,c} * PTT_m\right). \qquad \text{(P1)}$$

$$PQD.FX_{cc} = 0.0 \qquad \not\forall PQD0_{cc}. \qquad \text{(P1b)}$$

$$PQCD_{cag,h} * QCD_{cag,h} = \sum_{cces\$map\_cag\_c_{cag,ccess}} PQD_{cces} * \left(1 + TV_{cces}\right) * QCD2_{cag,cces,h} \qquad \text{(P2)}$$

$$PQCD.FX_{cc,h} = 0.0 \qquad \not\forall qcag_{cc,k}. \qquad \text{(P2b)}$$

$$PQS_c = \frac{PD_c * QD_c + PM_c * QM_c}{QQ_c} \qquad \forall cd \ \textbf{OR} \ cm. \qquad \text{(P3)}$$

$$PXC_c = \frac{PD_c * QD_c + \left(PE_c * QE_c\right)\$ce_c}{QXC_c} \qquad \forall cx. \qquad \text{(P4)}$$

$$PXC.FX_c = 0.0 \qquad \not\forall cx. \qquad \text{(P4b)}$$

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20 Using the properties of linearly homogenous functions defined by reference to Eulers theorem.
21 Using the properties of linearly homogenous functions defined by reference to Eulers theorem.

Domestic agents consume composite consumption commodities (*QQ*) that are aggregates of domestically produced and imported commodities. The prices of these composite commodities (*PQD*) are defined (P3) as the supply prices of the composite commodities plus *ad valorem* sales taxes (*TS*) and excise taxes (*TEX*) and the per unit cost of the margin services used in its delivery to consumers. It is relatively straightforward to include additional commodity taxes.

The prices of the aggregate commodities are defined as quantity weighted shares of the components of each aggregate (P4). There are four points to note in this relationship that derives from the application of Euler's theorem to linear homogenous functions. First, the prices of the natural commodities (*QCD*2) are only indexed on the natural commodity. Second, VAT taxes are levied on the natural commodities. Third, no VAT is levied on the aggregate commodity. And fourth, the weights are quantities and the quantities are variables and therefore change as the solution is determined.

Numéraire Price Block

The price block is completed by two price indices that can be used for price normalisation. Equation (N1) is for the consumer price index (*CPI*), which is defined as a weighted sum of composite commodity prices (*PQD*) in the current period, where the weights are the shares of each commodity in total demand (*comtotsh*). The domestic producer price index (*PPI*) is defined (N2) by reference to the supply prices for domestically produced commodities (*PD*) with weights defined as shares of the value of domestic output for the domestic market (*vddtotsh*).

Numéraire Block Equations

$$CPI = \sum_c comtotsh_c * \left(PQD_c + \left(1 + TV_c\right)\right). \qquad \text{(N1)}$$

$$PPI = \sum_c vddtotsh_c * PD_c \;. \qquad \text{(N2)}$$

Production Block

The supply prices of domestically produced commodities are determined by purchaser prices of those commodities on the domestic and international markets. Allowing for the possibility that the optimal output mix produced by an activity can vary according to the relative prices paid for the commodities produced by each activity means that the (weighted) average activity prices (*PX*) where the weights are quantities of each commodity produced by each activity (*IOQXACQX*).[22] The determination of the optimal mixes of commodities produced by each activity are detailed below (X19).

Production Block Equations: Top Level

$$PX_a = \sum_c IOQXACQX_{a,c} * PXC_c \ . \qquad \text{(X1)}$$

$$PX_a * (1 - TX_a) * QX_a = (PVA_a * QVA_a) + (PINT_a * QINT_a) \ . \qquad \text{(X2)}$$

$$PINT_a = \sum_c \left(ioqtdqd_{c,a} * PQD \right)_c \qquad \text{(X3)}$$

$$ADX_a = \left[(adxb_a + dabadx_a) * ADXADJ \right] + (DADX * adx01_a) \qquad \text{(X4)}$$

$$QX_a = AD_a^x \left(\delta_a^x QVA_a^{-rhoc_a^x} + (1 - \delta_a^x) QINT_a^{-rhoc_a^x} \right)^{-\frac{1}{rhoc_a^x}} \qquad \forall aqx_a \ . \qquad \text{(X5)}$$

$$\frac{QVA_a}{QINT_a} = \left[\frac{PINT_a}{PVA_a} * \frac{\delta_a^x}{(1 - \delta_a^x)} \right]^{\frac{1}{(1 + rhoc_a^x)}} \qquad \forall aqx_a \ . \qquad \text{(X6)}$$

$$QVA_a = ioqvaqx_a * QX_a \qquad \forall aqxn_a \qquad \text{(X7a)}$$

$$QINT_a = ioqintqx_a * QX_a \qquad \forall aqx_a \qquad \text{(X7b)}$$

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22 In the special case of each activity producing only one commodity **and** each commodity only being produced by a single activity, which is the case in the reduced form model reported in Dervis *et al*., (1982), then the aggregation weights *ioqxacqx* correspond to an identity matrix.

In this model a three-stage production process is adopted, with the top level as a CES or Leontief function. If a CES is imposed for an activity the value of activity output can be expressed as the volume share weighted sums of the expenditures on inputs after allowing for the production taxes (*TX*), which are assumed to be applied *ad valorem* (X1). This requires the definition of aggregate prices for intermediates (*PINT*); these are defined as the intermediate input-output coefficient weighted sum of the prices of intermediate inputs (X3), where $ioqtdqd_{c,a}$ are the intermediate input-output coefficients where the output is the aggregate intermediate input (*QINT*).

With CES technology the output by an activity, (*QX*) is determined by the aggregate quantities of factors used (*QVA*), i.e., aggregate value added, and aggregate intermediates used (*QINT*), where δ_a^x is the share parameter, $rhoc_a^x$ is the substitution parameter and AD_a^x is the efficiency variable (X5). Note how the efficiency/shift factor is defined as a variable and an adjustment mechanism is provided (X4), where *adxb* is the base values, *dabadx* is an absolute change in the base value, *ADXADJ* is an equiproportionate (multiplicative) adjustment factor, *DADX* is an additive adjustment factor and *adx*01 is a vector of zeros and non zeros used to scale the additive adjustment factor. The operation of this type of adjustment equation is explained below for the case of the import duty case. The associated the first order conditions defining the optimum ratios of value added to intermediate inputs can be expressed in terms of the relative prices of value added (*PVA*) and intermediate inputs (*PINT*), see (X6).

With Leontief technology at the top level the aggregate quantities of factors used (*QVA*), i.e., aggregate value added, and intermediates used (*QINT*), are determined by simple aggregation functions, (X7a) and (X7b), where *ioqvaqx* and *ioqintqx* are the (fixed) volume shares of *QVA* and *QINT* (respectively) in *QX*. The choice of top level aggregation function is controlled by the membership of the set *aqx*, with the membership of *aqxn* being the complement of *aqx*.

There are two arms to the second level production nest. For aggregate value added (*QVA*) the production function is a multi-factor CES function (X9) where δ_a^{va} is the share parameter, $rhoc_a^{va}$ is the substitution parameter and AD_a^{va} is the efficiency factor. The associated first order conditions for profit maximisation (X10) determine the wage rate of factors (*WF*), where the ratio of factor payments to factor *f* from activity *a* (*WFDIST*) are included to allow for non-homogenous factors, and is derived directly from the first order condition for profit maximisation as equalities between the wage rates for each factor in each activity and the values of the marginal products of those factors in each activity,[23] Again the efficiency/shift factor is defined as a variable with an adjustment mechanism (X8), where *advab* is the base values, *dabadva* is an absolute change in the base value, *ADVAADJ* is an equiproportionate (multiplicative) adjustment factor, *DADVA* is an additive adjustment factor and *adva*01 is a vector of zeros and non zeros used to scale the additive adjustment factor.

Production Block Equations: Second Level

$$ADVA_a = \left[\left(advab_a + dabadva_a\right) * ADVAADJ\right] + \left(DADVA * adva01_a\right) \qquad \text{(X8)}$$

$$QVA_a = AD_a^{va} * \left[\sum_{f\$\delta_{f,a}^{va}} \delta_{f,a}^{va} * ADFD_{f,a} * FD_{f,a}^{-\rho_a^{va}}\right]^{-1/\rho_a^{va}} \qquad \forall \rho_a^{va} \qquad \text{(X9)}$$

$$\begin{aligned} & WF_f * WFDIST_{f,a} * \left(1 + TF_{f,a}\right) \\ & = PVA_a * AD_a^{va} * \left[\sum_{f\$\delta_{f,a}^{va}} \delta_{f,a}^{va} * ADFD_{f,a} * FD_{f,a}^{-\rho_a^{va}}\right]^{-\left(\frac{1+\rho_a^{va}}{\rho_a^{va}}\right)} * \delta_{f,a}^{va} * FD_{f,a}^{\left(-\rho_a^{va}-1\right)} \\ & = PVA_a * QVA_a * AD_a^{va} * \left[\sum_{f\$\delta_{f,a}^{va}} \delta_{f,a}^{va} * ADFD_{f,a} * FD_{f,a}^{-\rho_a^{va}}\right]^{-1} \\ & \quad * \delta_{f,a}^{va} * ADFD_{f,a}^{-\rho_a^{va}} * \delta_{f,a}^{va} * FD_{f,a}^{\left(-\rho_a^{va}-1\right)} \qquad \forall \delta_{f,a}^{va} \text{ and } f2_{ff} \end{aligned} \qquad \text{(X10)}$$

$$WF.FX_{ff\,c} = WF0_{ff} \qquad \not\forall f_{ff}\,. \qquad \text{(X10b)}$$

$$QINTD_c = \sum_a ioqtdqd_{c,a} * QINT_a\,. \qquad \text{(X11)}$$

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23 The formulation in top line of (X10) implies that both the activity outputs (*QX*) and factor demands are solved simultaneously through the profit maximisation process. However, the formulation in the second line is more flexible since, *inter alia*, it allows the possibility of production rationing, i.e., activity outputs (*QX*) were fixed, but there was still cost minimisation. Thanks are due to Sherman Robinson for the explanation as to the theoretic and practical distinction between these alternative, but mathematically equivalent, formulations.

The third level production functions (X13) define the quantities of aggregate factors (*fag*) as CES aggregates of the labour factors (*l*). As elsewhere the efficiency factors ($ADFAG_{fag,a}$) and the factor shares ($\delta^{fd}_{fag,l,a}$) calibrated from the data and the elasticities of substitution, from which the substitution parameters are derived ($\rho^{fd}_{fag,a}$), are exogenously imposed. The matching first order conditions (X14) define the wage rate for a specific factor used by a specific activity as the average wage rate for that factor (WF_l) times a factor and activity specific factor 'efficiency' parameter ($WFDIST_{l,a}$); these ratios of payments to factor *l* from activity *a* are included to allow for non-homogenous factors where the differentiation is defined solely in terms of the activity that employs the factor. However the actual returns to a factor must be adjusted to allow for taxes on factor use ($TF_{l,a}$)

Production Block Equations: Third Level

$$ADFAG_{ff,a} = \left(adfagb_{ff,a} + dabfag_{ff,a}\right) + \left(ADFAGfADJ_{ff} * ADFAGaADJ_{a}\right) \qquad \text{(X12)}$$

$$FD_{ff,a} = AD^{fag}_{ff,a} * \left[\sum_{f\$\delta^{fd}_{ff,l,a}} \delta^{fd}_{ff,l,a} * FD_{l,a}^{-\rho^{fd}_{ff,a}}\right]^{-1/\rho^{fd}_{ff,a}} \qquad \forall FD0_{ff,a} \text{ and } fag_{ff} \quad \text{(X13)}$$

$$FD.FX_{f,a} = 0.0 \qquad \not\forall SAM_{f,a} \qquad \text{(X13b)}$$

$$\begin{aligned} &WF_l * WFDIST_{l,a} * \left(1 + TF_{l,a}\right) \\ &\quad = WF_{ff} * WFDIST_{ff,a} * \left(1 + TF_{ff,a}\right) * FD_{ff,a} \\ &\qquad * \left[\sum_{l\$\delta^{fd}_{ff,l,a}} \delta^{va}_{ff,l,a} * FD_{l,a}^{-\rho^{fd}_{ff,a}}\right]^{-1} * \delta^{va}_{ff,l,a} * FD_{l,a}^{\left(-\rho^{fd}_{ff,a}-1\right)} \qquad \forall \delta^{va}_{ff,l,a} \text{ and } fag_{ff} \end{aligned}$$

(X14)

The assumption of a three-stage production nest with Constant Elasticity of Substitution between aggregate intermediate input demand and aggregate value added and Leontief technology on intermediate inputs means that intermediate commodity demand (*QINTD*) is defined as the product of the fixed (Leontief) input coefficients of demand for commodity *c* by activity *a* (*ioqtdqd*), multiplied by the quantity of activity intermediate input (*QINT*) (X11).

The composite supplies of each commodity (*QXC*) are aggregates of the commodity outputs by each activity (*QXAC*). The default assumption is that when a commodity is produced by multiple activities it is differentiated by reference to the activity that produces the commodity; this is achieved by defining total production of a commodity as a CES aggregate of the quantities produced by each activity (X15). This provides a practical/modelling solution for two typical situations; first, where there are quality differences between two commodities that are notionally the same, e.g., modern digital vv disposable cameras, and second, where the mix of commodities within an aggregate differ between activities, e.g., a cereal grain aggregate made up of wheat and maize (corn) where different activities produce wheat and maize in different ratios. This assumption of imperfect substitution is implemented by a CES aggregator function with $adxc_c$ as the shift parameter, $\delta_{a,c}^{xc}$ as the share parameter and ρ_c^{xc} as the elasticity parameter.

The matching first order condition for the optimal combination of commodity outputs is therefore given by (X16), where *PXAC* are the prices of each commodity produced by each activity. Note how, as with the case of the value added production function two formulations are given for the first-order conditions and the second version is the default version used in the model. Further note that the efficiency/shift factor is in this case declared as a parameter; this reflects the expectation that there will be no endogenously determined changes in these shift factors.

However, there are circumstances where perfect substitution may be a more appropriate assumption given the characteristics of either or both of the activity and commodity accounts. Thus an alternative specification for commodity aggregation is proved where commodities produced by different activities are modeled as perfect substitutes, (X17), and the matching price condition is therefore requires that *PXAC* is equal to *PXC* for relevant commodity activity combinations (X18). The choice of aggregation function is controlled by the membership of the set *cxac*, with the membership of *cxacn* being the complement of *cxac*.

Finally, it is necessary to determine the quantities of each commodity produced by each activity. There are two basic assumptions included in the model: first that secondary commodities are produced with pure by-product technologies, i.e., in a fixed ratio to the principle product, and second that activities can adjust their output mix in response to changes in the prices of the commodities they produce. The function for by-product assumption is that fixed shares of products (*IOQXACQX*) are produced by each activity according to its level of total output (*QX*); although the shares are defined as variable the user determines which rows of the matrix *IOQXACQX* are fixed when configuring the model by defining membership of the set *acet* (X19). In order to implement the alternative assumption, it is only necessary to specify the first order condition for a CET function; this is reported in equation (X20). However, it is also now necessary to include a market clearing condition for production; this is reported in the market clearing section below (see equation C2).

Production Block Equations: Commodity Outputs

$$QXC_c = adxc_c * \left[\sum_{a\$\delta^{xc}_{a,c}} \delta^{xc}_{a,c} * QXAC_{a,c}^{-\rho^{xc}_c} \right]^{-1/\rho^{xc}_c} \qquad \forall cx_c \text{ and } cxac_c . \qquad \text{(X15)}$$

$$\begin{aligned} PXAC_{a,c} &= PXC_c * adxc_c * \left[\sum_{a\$\delta^{xc}_{a,c}} \delta^{xc}_{a,c} * QXAC_{a,c}^{-\rho^{xc}_c} \right]^{-\left(\frac{1+\rho^{xc}_c}{\rho^{xc}_c}\right)} * \delta^{xc}_{a,c} * QXAC_{a,c}^{\left(-\rho^{xc}_c - 1\right)} \\ &= PXC_c * QXC_c * \left[\sum_{a\$\delta^{xc}_{a,c}} \delta^{xc}_{a,c} * QXAC_{a,c}^{-\rho^{xc}_c} \right]^{-\left(\frac{1+\rho^{xc}_c}{\rho^{xc}_c}\right)} * \delta^{xc}_{a,c} * QXAC_{a,c}^{\left(-\rho^{xc}_c - 1\right)} \qquad \text{(X16)} \\ & \qquad \forall \delta^{xc}_{a,c} \text{ and } cxac_c \end{aligned}$$

$$PXAC.FX_{a,c} = 0.0 \qquad \not\forall SAM_{a,c} \qquad \text{(X16b)}$$

$$QXC_c = \sum_a QXAC_{a,c} \qquad \forall cx_c \text{ and } cxacn_c . \qquad \text{(X17)}$$

$$PXAC_{a,c} = PXC_c \qquad \forall \delta^{xc}_{a,c} \text{ and } cxacn_c . \qquad \text{(X18)}$$

$$QXAC_{a,c} = IOQXACQX_{a,c} * QX_a \qquad \forall IOQXACQX_{a,c} \textbf{ and } acetn_a . \qquad \text{(X19)}$$

$$QXAC_{a,c} = QX_a * \left(\frac{PXAC_{a,c}}{\left(PX_a * gamma_{a,c}^i * at_a^{i\,\rho_a^i} \right)} \right)^{\left(1/\left(\rho_a^i - 1\right) \right)} \qquad \text{(X20)}$$

$\forall IOQXACQX_{a,c}$ **and** $acet_a$

$$QXAC.FX_{a,c} = 0.0 \qquad \forall SAM_{a,c} \qquad \text{(X20b)}$$

<u>Factor Block</u>

There are two sources of income for factors. First there are payments to factor accounts for services supplied to activities, i.e., domestic value added, and second there are payments to domestic factors that are used overseas, the value of these are assumed fixed in terms of the foreign currency. Factor incomes (*YF*) are therefore defined as the sum of all income to the factors across all activities (F1)

Factor Block Equations

$$YF_f = \left(\sum_a WF_f * WFDIST_{f,a} * FD_{f,a} \right) + \left(factwor_f * ER \right). \qquad \text{(F1)}$$

$$YFDISP_f = \left(YF_f * \left(1 - deprec_f \right) \right) * \left(1 - TYF_f \right). \qquad \text{(F2)}$$

$$YFINS_f = YFDISP_f - \left[\left(govvash_f * YFDISP_f \right) + \left(worvash_f * YFDISP_f \right) \right]. \qquad \text{(F3)}$$

$$FSISH_{insw,f} = FSI_{insw,f} \Big/ \sum_{insw} FSI_{insw,f} \qquad \text{(F4)}$$

$$FSISH.FX_{insw,f} = 0.0 \qquad \not\forall FSISH0_{insw,f} \qquad \text{(F4b)}$$

$$INSVA_{insw,f} = FSISH_{insw,f} * YFINS_f \qquad \text{(F5)}$$

$$INSVA.FX_{insw,f} = 0.0 \qquad \not\forall INSVA0_{insw,f} \qquad \text{(F5b)}$$

Before distributing factor incomes to the institutions that supply factor services allowance is made for depreciation rates (*deprec*) and factor (income) taxes (*TYF*) so that factor income for distribution (*YFDISP*) is defined (F2).

The endogenous determination of factor incomes requires the definition of variables that control that distribution. The key assumption is that the shares of factor income (*FSISH*) distributed to institutions (*insw*) are defined by the shares of factor ownership (*FSI*), which is implemented in (F3). For coding convenience the values of factor incomes distributed to each institution (*INSVA*) are calculated explicitly (F4); this reduces the code needed later although it increases the number of variables in the model.

Household Block

Household Income

Households receive income from a variety of sources (H1). Factor incomes are distributed to households in proportion to their ownership of factors ($INSVA_{h,f}$), plus inter household transfers (*HOHO*), distributed payments/dividends from incorporated enterprises (*HOENT*) and real transfers from government (*hogovconst*) that are adjustable using a scaling factor (*HGADJ*) and transfers from the rest of the world (*howor*) converted into domestic currency units.

Household Expenditure

Inter household transfers (*HOHO*) are defined (H2) as a fixed proportions of household income (*YH*) after payment of direct taxes and savings, and then household consumption expenditure (*HEXP*) is defined as household income after tax income less savings and transfers to other households (H3).

Households are then assumed to maximise utility subject to nested CES and Stone-Geary (*aka* LES) utility functions. In a Stone-Geary utility function household consumption demand consists of two components; ‘subsistence’ demand (*qcdconst*) and ‘discretionary’ demand, and the equation must therefore capture both elements. This is written as two equation, (H4) and (H5), where the first encompasses the natural commodities that are independent arguments in the utility function and the second encompasses the aggregate commodities. Discretionary demand is then defined as the marginal budget shares (*beta*) spent on each commodity out of ‘uncommitted’ income, i.e., household consumption expenditure less total expenditure on ‘subsistence’ demand of BOTH natural and aggregate commodities. In this system of nested utility functions, the commodities in the LES utility function are (typically) defined as ‘broad’ commodity groups, e.g., food, clothing, utilities, etc., that are aggregates of ‘natural’ commodities or ‘natural’ commodities that are deemed sufficiently distinctive as to justify the assumption that they are characterised by having a distinct level of ‘subsistence’ demand. The set *cles*(*cc*) is therefore defined to encompass all commodities, natural or aggregated, that enter into the LES utility functions, and the LES function is calibrated accordingly. If the user wants to assume Cobb-Douglas utility functions, at the top level, for one or more households this can be achieved by setting the Frisch parameters equal to minus one and all the income elasticities of demand equal to one (the model code includes documentation of the calibration steps). This is typically only the case for relatively rich households where the operation of the utility function will not reduce demand below a level consistent with subsistence demand.

The second level of the utility functions is defined with CES preferences. The quantities of the aggregated commodity groups that are demanded by each household ($QCD_{cag,,h}$) are defined in the top level (LES) utility function and therefore only the first order conditions are required to determine the optimum combinations of natural commodities. This is presented as a standard FOC for a CES function which has been calibrated for shift, share and elasticity parameters based on the initial data and the, exogenous imposed, substitution elasticities that are aggregate commodity and household specific.

Household Income and Expenditure Block Equations

$$YH_h = \left(\sum_f INSVA_{h,f} \right) + \left(\sum_{hp} HOHO_{h,hp} \right) + HOENT_h + \left(hogovconst_h * HGADJ * CPI \right) + \left(howor_h * ER \right) . \qquad \text{(H1)}$$

$$HOHO_{h,hp} = hohosh_{h,hp} * \left(YH_h * \left(1 - TYH_h \right) \right) * \left(1 - SHH_h \right) . \qquad \text{(H2)}$$

$$HOHO_{h,hp} = 0.0 \qquad \not\forall SAM_{h,hp}. \tag{H2b}$$

$$HEXP_h = \left(\left(YH_h * \left(1 - TYH_h\right)\right) * \left(1 - SHH_h\right)\right) - \left(\sum_{hp} HOHO_{hp,h}\right). \tag{H3}$$

Household Utility Function Block Equations

$$QCD_{cc,h} = \frac{\left(\sum_h \left(\begin{array}{l}\left(PQD_{cc} * \left(1 + TV_{cc}\right) * qcdconst_{cc,h}\right) + \sum_h beta_{cc,h} \\ * \left(\begin{array}{l} HEXP_h - \sum_c \left(PQCD_{cag,h} * qcdconst_{cag,h}\right) \\ - \sum_c \left(PQD_{cc} * \left(1 + TV_{cc}\right) * qcdconst_{cc,h}\right)\end{array}\right)\end{array}\right)\right)}{\left(PQD_{cc} * \left(1 + TV_{cc}\right)\right)} \tag{H4}$$

$$\forall ccesn(cc)$$

$$QCD.FX_{cc,h} = 0.0 \qquad \not\forall beta_{cc,h} \tag{H4b}$$

$$QCD_{cag,h} * PQCD_{cagc} = \left(qcdconst_{cc,h} * PQCD_{cc}\right) + beta_{cag,h} * \left(\begin{array}{l} HEXP_h - \sum_{cag} \left(PQCD_{cag,h} * qcdconst_{cag,h}\right) \\ - \sum_{ccesn} \left(PQD_{ccesn} * \left(1 + TV_{ccesn}\right) * qcdconst_{ccesn,h}\right)\end{array}\right) \tag{H5}$$

$$QCD2_{cag,cc,h} = QCD_{cag,h} * \left(\frac{\left(\left(PQD_{cc} * \left(1 + TV_{cc}\right)\right) * accd_{cag,h}{}^{-\rho^{cd}_{cag,h}}\right)}{\left(PQCD_{cag,h} * \delta^{cd}_{cag,cc,h}\right)}\right)^{\left(-1 / \left(\rho^{cd}_{cag,h} + 1\right)\right)} \tag{H6}$$

$$\forall cces(cc)$$

$$QCD2.FX_{cc,cc,h} = 0.0 \qquad \not\forall \delta^{cd}_{cag,cc,h} \tag{H6}$$

Enterprise Block

Enterprise Income

Similarly, income to enterprises (EN1) comes from the share of distributed factor incomes accruing to enterprises ($INSVA_{e,f}$) and real transfers from government (*entgovconst*) that are adjustable using a scaling factor (*EGADJ*) and the rest of the world (*entwor*) converted in the domestic currency units.

Enterprise Block Equations

$$YE_e = \left(\sum_f INSVA_{e,f}\right) + \left(entgovconst_e * EGADJ * CPI\right) + \left(entwor_e * ER\right). \quad \text{(EN1)}$$

$$QED_{c,e} = qedconst_{c,e} * QEDADJ. \quad \text{(EN2)}$$

$$HOENT_{h,e} = hoentsh_{h,e} * \begin{pmatrix} \left(YE_e * \left(1 - TYE_e\right)\right) * \left(1 - SEN_e\right) \\ -\sum_c \left(QED_{c,e} * PQD_c\right) \end{pmatrix} \quad \text{(EN3)}$$

$$GOVENT_e = goventsh_e * \begin{pmatrix} \left(YE_e * \left(1 - TYE_e\right)\right) * \left(1 - SEN_e\right) \\ -\sum_c \left(QED_c * PQD_c\right) \end{pmatrix} \quad \text{(EN4)}$$

$$VED_e = \left(\sum_c QED_{c,e} * PQD_c\right). \quad \text{(EN5)}$$

Enterprise Expenditure

The consumption of commodities by enterprises (*QED*) is defined (EN2) in terms of fixed volumes (*qedconst*), which can be varied via the volume adjuster (*QEDADJ*), and associated with any given volume of enterprise final demand there is a level of expenditure (*VED*); this is defined by (EN5) and creates an option for the macroeconomic closure conditions that distribute absorption across domestic institutions (see below).

If *QEDADJ* is made flexible, then *qedconst* ensures that the quantities of commodities demanded are varied in fixed proportions; clearly this specification of demand is not a consequence of a defined set of behavioural relationships, as was the case for households, which reflects the difficulties inherent to defining utility functions for non-household institutions. If *VED* is fixed then the volume of consumption by enterprises (*QED*) must be allowed to vary, via the variable *QENTDADJ*.

The incomes to households from enterprises, which are assumed to consist primarily of distributed profits/dividends, are defined by (EN3), where *hoentsh* are defined as fixed shares of enterprise income after payments of direct/income taxes, savings and consumption expenditure. Similarly the income to government from enterprises, which is assumed to consist primarily of distributed profits/dividends on government owned enterprises, is defined by (EN4), where *goventsh* is defined as a fixed share of enterprise income after payments of direct/income taxes, savings and consumption expenditure.

Government Block

Tax Rates

All tax rates are variables in this model. The tax rates in the base solution are defined as parameters, e.g., tmb_c are the import duties by commodity *c* in the base solution, and the equations then allow for varying the tax rates in 4 different ways. For each tax instrument there are four methods that allow adjustments to the tax rates; two of the methods use variables that can be solved for optimum values in the model according to the choice of closure rule and two methods allow for deterministic adjustments to the structure of the tax rates. The operation of this method is discussed in detail only for the equations for import duties while the other equations are simply reported.

Import duty tax rates are defined by (GT1), where tmb_c is the vector of import duties in the base solution, $dabtm_c$ is a vector of absolute changes in the vector of import duties, *TMADJ* is a variable whose initial value is ONE, *DTM* is a variable whose initial value is ZERO and $tm01_c$ is a vector of zeros and non zeros. In the base solution the values of $tm01_c$ and $dabtm_c$ are all ZERO and *TMADJ* and *DTM* are fixed as their initial values – a closure rule decision –then the applied import duties are those from the base solution. Now the different methods of adjustment can be considered in turn

1. If *TMADJ* is made a variable, which requires the fixing of another variable, and all other initial conditions hold then the solution value for *TMADJ* yields the optimum equiproportionate change in the import duty rates necessary to satisfy model constraints, e.g., if *TMADJ* equals 1.1 then all import duties are increased by 10%.
2. If any element of *dabtm* is non zero and all the other initial conditions hold, then an absolute change in the initial import duty for the relevant commodity can be imposed using *dabtm*, e.g., if *tmb* for one element of *c* is 0.1 (a 10% import duty) and *dabtm* for that element is 0.05, then the applied import duty is 0.15 (15%).
3. If *TMADJ* is a variable, any elements of *dabtm* are non zero and all other initial conditions hold then the solution value for *TMADJ* yields the optimum equiproportionate change in the applied import duty rates.
4. If *DTM* is made a variable, which requires the fixing of another variable, AND at least one element of *tm*01 is NOT equal to ZERO then the subset of elements of *c* identified by *tm*01 are allowed to (additively) increase by an equiproportionate amount determined by the solution value for *DTM* times the values of *tm*01. Note how in this case it is necessary to both 'free' a variable and give values to a parameter for a solution to emerge.

This combination of alternative adjustment methods covers a range of common tax rate adjustment used in many applied applications while being flexible and easy to use.

Export tax rates are defined by (GT2), where tme_c is the vector of export duties in the base solution, $dabte_c$ is a vector of absolute changes in the vector of export duties, *TEADJ* is a variable whose initial value is ONE, *DTE* is a variable whose initial value is ZERO and $te01_c$ is a vector of zeros and non zeros. Sales tax rates are defined by (GT3), where tms_c is the vector of sales tax rates in the base solution, $dabts_c$ is a vector of absolute changes in the vector of sales taxes, *TSADJ* is a variable whose initial value is ONE, *DTS* is a variable whose initial value is ZERO and $ts01_c$ is a vector of zeros and non zeros. And excise tax rates are defined by (GT6), where $texb_c$ is the vector of excise tax rates in the base solution, $dabtex_c$ is a vector of absolute changes in the vector of import duties, *TEXADJ* is a variable whose initial value is ONE, *DTEX* is a variable whose initial value is ZERO and $tex01_c$ is a vector of zeros and non zeros. The model also allows for specific, quantity rather than value based, taxes on commodities (*TQS*) in equation GT4, and value added taxes (*TV*) in equation GT5.

Tax Rate Block Equations

$$TM_c = \left(\left(tmb_c + dabtm_c \right) * TMADJ \right) + \left(DTM * tm01_c \right) \quad \text{(GT1)}$$

$$TE_c = \left(\left(teb_c + dabte_c \right) * TEADJ \right) + \left(DTE * te01_c \right) \quad \text{(GT2)}$$

$$TS_c = \left(\left(tsb_c + dabts_c \right) * TSADJ \right) + \left(DTS * ts01_c \right) \quad \text{(GT3)}$$

$$TV_c = \left(\left(tvb_c + dabtv_c \right) * TVADJ \right) + \left(DTV * tv01_c \right) \quad \text{(GT4)}$$

$$TV.FX_c = 0.0 \qquad \forall c_{cc} \quad \text{(GT4b)}$$

$$TEX_c = \left(\left(texb_c + dabtex_c \right) * TEXADJ \right) + \left(DTEX * tex01_c \right) \quad \text{(GT5)}$$

$$TX_a = \left(\left(txb_a + dabtx_a \right) * TXADJ \right) + \left(DTX * tx01_a \right) \quad \text{(GT6)}$$

$$TF_{f,a} = \left(\left(tfb_{f,a} + dabtf_{f,a} \right) * TFADJ \right) + \left(DTF * tf01_{f,a} \right) \quad \text{(GT7)}$$

$$TF.FX_{ff,a} = TF0_{ff,a} \qquad \forall f_{ff} \quad \text{(GT7b)}$$

$$TYF_f = \left(\left(tyfb_f + dabtyf_f \right) * TYFADJ \right) + \left(DTYF * tyf01_f \right) \quad \text{(GT8)}$$

$$TYH_h = \left(\left(tyhb_h + dabtyh_h \right) * TYHADJ \right) + \left(DTYH * tyh01_h \right) \quad \text{(GT9)}$$

$$TYE_e = \left(\left(tyeb_e + dabtye_e \right) * TYEADJ \right) + \left(DTYE * tye01_e \right). \quad \text{(GT10)}$$

Indirect tax rates on production are defined by (GT7), where txb_c is the vector of production taxes in the base solution, $dabtx_c$ is a vector of absolute changes in the vector of production taxes, *TXADJ* is a variable whose initial value is ONE, *DTX* is a variable whose initial value is ZERO and $tx01_c$ is a vector of zeros and non zeros. Taxes on factor use by each factor and activity are defined by (GT8), where $tfb_{f,a}$ is the matrix of factor use tax rates in the base solution, $dabtf_{f,a}$ is a matrix of absolute changes in the matrix of factor use taxes, *TFADJ* is a variable whose initial value is ONE, *DTFM* is a variable whose initial value is ZERO and $tf01_{f,a}$ is a matrix of zeros and non zeros.

Factor income tax rates[24] are defined by (GT9), where $tyfb_f$ is the vector of factor income taxes in the base solution, $dabtyy_f$ is a vector of absolute changes in the vector of factor income taxes, *TYFADJ* is a variable whose initial value is ONE, *DTYF* is a variable whose initial value is ZERO and $tyf01_f$ is a vector of zeros and non zeros. Household income tax rates are defined by (GT10), where $tyhb_h$ is the vector of household income tax rates in the base solution, $dabtyh_h$ is a vector of absolute changes in the vector of income tax rates, *TYFADJ* is a variable whose initial value is ONE, *DTYF* is a variable whose initial value is ZERO and $tyh01_c$ is a vector of zeros and non zeros. And finally, enterprise income tax rates are defined by (GT11), where $tyeb_e$ is the vector of enterprise income tax rates in the base solution, $dabtye_e$ is a vector of absolute changes in the income tax rates, *TYEADJ* is a variable whose initial value is ONE, *DTYE* is a variable whose initial value is ZERO and $tye01_e$ is a vector of zeros and non zeros.

Tax Revenues

Although it is not necessary to keep the tax revenue equations separate from other equations, e.g., they can be embedded into the equation for government income (*YG*), it does aid clarity and assist with implementing fiscal policy simulations. For this model there are eight tax revenue equations. The patterns of tax rates are controlled by the tax rate variable equations. In all cases the tax rates can be negative indicating a ‘transfer’ from the government.

There are four tax instruments that are dependent upon expenditure on commodities, with each expressed as an *ad valorem* tax rate. Tariff revenue (*MTAX*) is defined (GR1) as the sum of the product of tariff rates (*TM*) and the value of expenditure on imports at world prices, the revenue from export duties (*ETAX*) is defined (GR2) as the sum of the product of export duty rates (*TE*) and the value of expenditure on exports at world prices, the sale tax revenues (*STAX*) are defined (GR3) as the sum of the product of sales tax rates (*TS*) and the value of domestic expenditure on commodities, and excise tax revenues (*EXTAX*) are defined (GR5) as the sum of the product of excise tax rates (*TEX*) and the value of domestic expenditure on commodities. The model also allows for specific, quantity rather than value based, taxes (GR4) on commodities (*QSTAX*) where the base for the tax is the quantity of the

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24 These are defined as taxes on factor incomes that are independent of the activity that employs the factor. They could include social security type payments.

commodity rather than value. There is also provision for value added tax (GR5) where as opposed to other taxes and commodities demanded domestically the tax is only paid on final demand by households.

Government Tax Revenue Block Equations

$$MTAX = \sum_{c}\left(TM_{c} * PWM_{c} * ER * QM_{c}\right). \qquad \text{(GR1)}$$

$$ETAX = \sum_{c}\left(TE_{c} * PWE_{c} * ER * QE_{c}\right). \qquad \text{(GR2)}$$

$$STAX = \sum_{c}\left(TS_{c} * PQS_{c} * QQ_{c}\right). \qquad \text{(GR3)}$$

$$EXTAX = \sum_{c}\left(TEX_{c} * PQS_{c} * QQ_{c}\right). \qquad \text{(GR4)}$$

$$\begin{aligned} VTAX = & \sum_{h}\sum_{c}\left(TV_{c} * PQD_{c} * QCD_{c,h\$ccesn_{c}}\right) \\ & + \sum_{cag}\sum_{h}\sum_{c}\left(TV_{c} * PQD_{c} * QCD2_{cag,c,h\$cces_{c}}\right) \end{aligned} \qquad \text{(GR5)}$$

$$ITAX = \sum_{a}\left(TX_{a} * PX_{a} * QX_{a}\right). \qquad \text{(GR6)}$$

$$FTAX = \sum_{f,a}\left(TF_{f,a} * WF_{f} * WFDIST_{f,a} * FD_{f,a}\right). \qquad \text{(GR7)}$$

$$FYTAX = \sum_{f}\left(TYF_{f} * \left(YF_{f} * \left(1 - deprec_{f}\right)\right)\right). \qquad \text{(GR8)}$$

$$DTAX = \sum_{h}\left(TYH_{h} * YH_{h}\right) + \sum_{e}\left(TYE_{e} * YE\right). \qquad \text{(GR9)}$$

There is a single tax on production (*ITAX*). As with other taxes this is defined (GR7) as the sum of the product of indirect tax rates (*TX*) and the value of output by each activity evaluated in terms of the activity prices (*PX*). In addition activities can pay taxes based on the value of employed factors – factor use taxes (FTAX). The revenue from these taxes is defined sum of the product of factor income tax rates and the value of the factor services employed by each activity for each factor; the sum is over both activities and factors. These two taxes are the instruments most likely to yield negative revenues through the existence of production and/or factor use subsidies.

Income taxes are collected on both factors and domestic institutions. The income tax on factors (*FYTAX*) is defined (GR9) as the product of factor tax rates (*TYF*) and factor incomes for all factors, while those on institutions (DTAX) are defined (GR10) as the sum of the product of household income tax rates (*TYH*) and household incomes plus the product of the direct tax rate for enterprises (*TYE*) and enterprise income.

Government Income

The sources of income to the government account (G1) are more complex than for other institutions. Income accrues from 9 tax instruments; tariff revenues (*MTAX*), export duties (*ETAX*), value added taxes (VTAX), (general) sales taxes (*STAX*), excise taxes (*EXTAX*), production taxes (*ITAX*), factor use taxes (*FTAX*), factor income taxes (*FYTAX*) and direct income taxes (*DTAX*), which are defined in the tax equation block above. In addition the government can receive income from its ownership of factors (*INSVA*), distributed payments/dividends from incorporated enterprises (*GOVENT*) and transfers from abroad (*govwor*) converted in the domestic currency units. It would be relatively easy to subsume the tax revenue equations into the equation for government income, but they are kept separate to facilitate the implementation of fiscal policy experiments. Ultimately however the choice is a matter of personal preference.

Government Expenditure Block

The demand for commodities by the government for consumption (*QGD*) is defined (G2) in terms of fixed proportions (*qgdconst*)[25] that can be varied with a scaling adjuster (*QGDADJ*), and associated with any given volume of government final demand there is a level/value of expenditure (*VGD*) defined by (G3); this creates an option for the macroeconomic closure conditions that distribute absorption across domestic institutions (see below).

[25] Alternative utility functions could be specified, e.g., Cobb-Douglas, CES, etc., but there is no substantive body of economic theory upon which such utility functions can be based. Hence the presumption of Leontief/fixed coefficient preference is a pragmatic, if simplistic, specification

Government Income and Expenditure Block Equations

$$\begin{aligned} YG = {} & MTAX + ETAX + STAX + EXTAX + VTAX \\ & + FTAX + ITAX + FYTAX + DTAX \\ & + \left(\sum_f INSVA_{g,f} \right) + GOVENT + \left(govwor * ER \right) \end{aligned} . \qquad \text{(G1)}$$

$$QGD_c = qgdconst_c * QGDADJ . \qquad \text{(G2)}$$

$$VGD = \left(\sum_c QGD_c * PQD_c \right) . \qquad \text{(G3)}$$

$$\begin{aligned} EG = {} & \left(\sum_c QGD_c * PQD_c \right) + \left(\sum_h hogovconst_h * HGADJ * CPI \right) \\ & + \left(\sum_e entgovconst_e * EGADJ * CPI \right) \end{aligned} \qquad \text{(G4)}$$

Hence, total government expenditure (*EG*) can be defined (G4) as equal to the sum of expenditure by government on consumption demand at current prices, plus real transfers to households (*hogovconst*) that can be adjusted using a (multiplicative) scaling factor (*HGADJ*) and real transfers to enterprises (*entgovconst*) that can also be adjusted by a (multiplicative) scaling factor (*EGADJ*).

As with enterprises there are difficulties inherent to defining utility functions for a government. Changing *QGDADJ*, either exogenously or endogenously, by allowing it to be a variable in the closure conditions, provides a means of changing the behavioural assumption with respect to the ‘volume’ of commodity demand by the government. If the value of government final demand (*VGD*) is fixed then government expenditure is fixed and hence the volume of consumption by government (*QGD*) must be allowed to vary, via the *QGDADJ* variable. If it is deemed appropriate to modify the patterns of commodity demand by the government then the components of *qgdconst* must be changed.

Kapital Block

Savings Block

The savings rates for households (*SHH* in I1) and enterprises (*SEN* in I2) are defined as variables using the same adjustment mechanisms used for tax rates; $shhb_h$ and $senb_e$ are the savings rates in the base solution, $dabshh_h$ and $dabsen_e$ are absolute changes in the base rates, *SHADJ* and *SEADJ* are multiplicative adjustment factors, *DSHH* and *DSEN* are additive adjustment factors and $shh01_h$ and $sen01_e$ are vectors of zeros and non zeros that scale the additive adjustment factors. However, unlike the tax rate equations, each of the savings rates equations has two additional adjustment factors – *SADJ* and *DS*. These serve to allow the user to vary the savings rates for households and enterprises in tandem; this is useful when the macroeconomic closure conditions require increases in savings by domestic institutions and it is not deemed appropriate to force all the adjustment on a single institution or group of institutions.[26]

Total savings in the economy are defined (I3) as shares (*SHH*) of households' after tax income, where direct taxes (*TYH*) have first call on household income, plus the allowances for depreciation at fixed rates (*deprec*) out of factor income, the savings of enterprise savings at fixed rates (*SEN*) out of after tax income, the government budget deficit/surplus (*KAPGOV*) and the current account 'deficit' (*CAPWOR*). The last two terms of I3 – *KAPGOV* and *CAPWOR* - are defined below by equations in the market clearing block.

Kapital Block Equations

$$SHH_h = \left(\left(shhb_h + dabshh_h\right) * SHADJ * SADJ\right) + \left(DSHH * DS * shh01_h\right) \tag{I1}$$

$$SEN_e = \left(\left(sen_e + dabsen_e\right) * SEADJ * SADJ\right) + \left(DSEN * DS * sen01_e\right) \tag{I2}$$

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[26] A similar mechanism can be easily imposed for tax rates when the user wishes to cause two or more tax instruments to move in tandem.

$$TOTSAV = \sum_h \left(\left(YH_h * \left(1 - TYH_h\right)\right) * SHH_h \right) + \sum_e \left(\left(YE * \left(1 - TYE_e\right)\right) * SEN_e \right) + \sum_f \left(YF_f * deprec_f \right) + KAPGOV + \left(CAPWOR * ER \right) \quad \text{(I3)}$$

$$QINVD_{c,i} = \left(QINV_i * ioqinvd_{c,i} \right). \quad \text{(I4)}$$

$$QINVD.FX_{c,i} = 0.0 \qquad \not\forall\, ioqinvd_{c,i}. \quad \text{(I4b)}$$

$$QINV_i = qinvb_i * IADJ . \quad \text{(I5)}$$

$$INVEST * INVSH\_I_i = \sum_c PQD_c * QINVD_{c,i} . \quad \text{(I6)}$$

$$INVEST = \sum_{c,i} PQD_c * QINVD_{c,i} . \quad \text{(I7)}$$

Investment Block

The presence of multiple types of capital goods implies the existence of different patterns of investment expenditures each corresponding to one type of capital good, where the set *i* defines the investment patterns and each member of *i* is uniquely paired with a member of the set *k* (capital factors).[27] In essence, this implies that each capital good has a unique cost of production (a form of 'production function') defined by the quantities of each commodity required to produce a given quantity of the capital good. If the functional form for the 'production functions' are assumed to be Leontief it is possible to derive input-output coefficients that define the quantities of each input/commodity required to produce a unit of each capital good ($ioqinvd_{c,i}$).[28] Then the demand for each commodity used to produce investment goods ($QINVD_{c,i}$) as the product of the quantity/volume of each capital good ($QINV_i$) and the respective coefficients ($ioqinvd_{c,i}$); these relationships are defined in I4 and I5.

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27 Note that *i* can be a single member set so the same code can be used where there is only a single investment account.

28 Alternative production functions can be easily specified, e.g., Cobb-Douglas, CES, etc., although information about the ease of substituting inputs in the production of capital goods is limited. Hence the presumption of Leontief technologies as a pragmatic, if simplistic, specification.

The demand for commodities for investment purposes therefore depends on the 'technologies' and the volumes of capital goods required. However, comparative static and recursive dynamic CGE models do not have endogenously determined investment functions that serve to define the quantities of capital goods produced. A simple, and very commonly used, dichotomy is to assume that the demand for capital goods is determined by either the amount of available investable funds – so-called savings driven assumption - or exogenously – so-called investment driven or Keynesian assumption. Assume, for purposes of exposition only, that an investment driven assumption of exogenous determination is appropriate, i.e., $QINV_i$ is fixed exogenously.

To implement the investment driven assumption, the parameters $qinvb_i$ are fixed, at the exogenously determined levels, and the scaling variable *IADJ* is fixed equal to one (I5); and hence the demand for each commodity for investment purposes are determined from I4. Given the demand for each commodity and the prices for each commodity (PQD_c) the value of investment expenditures to produce each capital good *i* ($INVEST*INVSH\_I_i$) is the product of prices and quantities (I6). The total value of all investments (*INVEST*) is then defined as the summation of the expenditures on each capital good (I7), and market clearing for investable funds is ensure by the equality of total savings (*TOTSAV*) and investment (see C20 below).

If a savings driven assumption is adopted, then the value of *INVEST* is determined by total savings (*TOTSAV*) and the parameters $qinvb_i$ determine the ratio of capital goods produced with the scaling variable *IADJ* providing (multiplicative) equiproportionate changes in the volumes of each capital good. The scaling variable *IADJ* adjusts the volumes of capital goods produced so that the expenditures on each capital good ($INVEST*INVSH\_I_i$) exhaust the available investable funds (*INVEST*); thus in such a setting I7 operates as a market clearing equation.

The members of the set *i* include the agent that gathers together investable funds from savings by domestic and foreign agents ('*i_s*') and distributes those funds across different investment activities. One such investment activity is stock changes ('*dstoc*'); thus stock changes can be included within this formulation.

In a comparative static context, the specification of different patterns of investment expenditures is relevant *if and only if* the analyst has information that indicates that the average pattern of investment expenditures will change due to the simulation. If the relative volumes of investment in capital goods is invariant, then the system operates as if there is a single investment account, i.e., the system *de facto* collapses back to the 'standard' approach in the STAGE family of models.

Foreign Institutions Block

The economy also employs foreign owned factors whose services must be recompensed. It is assumed that these services receive proportions of the factor incomes available for distribution, (W1).

Foreign Institutions Block Equations

$$YFWOR_f = \sum_w INSVA_{w,f} \quad . \qquad \text{(W1)}$$

Market Clearing Block

The market clearing equations ensure the simultaneous clearing of all markets. In this model there are six relevant markets: factor and commodity markets and enterprise, government, capital and rest of world accounts. Market clearing with respect to activities has effectively been achieved by (X16), wherein the supply and demand for domestically produced commodities was enforced, while the demand system and the specification of expenditure relationships ensures that the household markets are cleared.

The description immediately below refers to a default set of closure rules/market clearing conditions for this model; a subsequent section explores alternative closure rules//market clearing configurations available with this model.

Factor Market Clearing

Adopting an initial assumption of full employment, which the model closure rules will demonstrate can be relaxed, amounts to requiring that the factor market is cleared by equating factor demands (*FD*) and factor supplies by institutions (*FSI*) for all factors (C1), and the supplies of each factor (*FS*) equal factor supplies by institutions (*FSI*) for all factors (C2).

Importantly, the factors (labour) used by institutions to produce leisure can only be supplied by the specific institution. Thus the factor quantities supplied by each institution for the production of leisure (*FSIL*) must be defined so as to be activity, and its paired RHG, and factor specific This is defined in equation C3 where the mapping (*map_hh_alei*) pairs leisure activities (*alei*) with RHGs (*hh*). Then the market clearing condition for the factor supplies by institution (*FSI*) and the demand for factors by non-leisure activities (*FD*) and leisure activities (*FSIL*) are determined as residuals.

Factor Market Clearing Block Equations

$$\sum_{insw} FSI_{insw,f} = \sum_{a} FD_{f,a} \quad . \tag{C1}$$

$$FS_{f} = \sum_{ins} FSI_{ins,f} \tag{C2}$$

$$FS.FX_{f} = FS0_{ff} \qquad \forall f_{ff} \tag{C2b}$$

$$FSIL_{insw,f} = \sum_{alei\$map\_hh\_alei_{ins,alei}} FD_{f,alei} \quad . \tag{C3}$$

There is however no reason to suppose that the proportionate changes in the amount of labour time devoted to leisure and non-leisure activities will be identical across households. Even if the elasticities controlling the operation of the RHGs utility functions are the same there are differences in the levels of household incomes and preferences, i.e., there will be differences in the shift and share parameters of the utility functions. Thus the presence of a labour-leisure trade-off means that the labour/factor supplies by institution (*FSI*) will be endogenously determined variables and hence the functional distribution of income can change. This achieved in equations (F3) and (F4).

Institution and Household Migration

A typical approach to modelling representative household groups (RHG), one of the groups that make up institutions, is to assume that households are rigidly segmented and that each RHG receives a fixed share of factor incomes generated domestically and received from abroad. The first part of the assumption requires that households are not allowed to migrate, e.g., rural-urban migration is precluded,[29] while the second part carries the implicit presumption that any changes in factor supplies by RHGs changes the supply of each factor by each RHG equiproportionately. Neither of these assumptions is necessarily an accurate reflection of reality and, at the same time, imposes restrictions that require households to NOT change behaviours in response to economic signals. This model allows RHGs to relocate/migrate in response to changes in economic signals, e.g., changes in relative RHG incomes, and, when they do so, to transfer their factors from the RHG they leave to the RHG they join, i.e., modifying the factor supplies by households (*FSI*) and hence the functional distribution of income. The model is coded so that the functional distribution of income changes as RHGs migrate; the logic of the code is the same as that used to allow factor mobility functions (see below).

The inclusion of the assumption of household migration across types of household relaxes the restrictive assumption of rigid segmentation of RHGs. The behavioural assumption is that the incentives for a RHG to migrate in the model are changes in the relative returns to different RHGs, i.e., only economic incentives are embodied within the behavioural assumption.[30] Thus if RHGs in one segment experiences a RELATIVE increase in income to the RHGs in all other segments there will be incentives for households to relocate/migrate to the RHG that receives an increase in RELATIVE income: relative incomes (*YMIGR*) are defined in (C4). There are two things about this relationship that are important to note: first it is assumed that all other influences on the location decisions of RHGs are unchanged, and second that the user needs to specify the set (*map_insw_inswp*) that defines the pairs of RHGs between which households can migrate.[31]

Institution Migration Block Equations

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[29] Thus while structural change through the patterns of production, trade, consumption and employment are simulated, households are not allowed to respond to incentives to relocate. Even in the context of a comparative static model this is a restrictive assumption, whereas in the context of a dynamic model it is arguably highly unrealistic.

[30] This arguably sufficient in a comparative static context since it would refer to marginal changes. Over the longer term it can be argued that this restriction ignores other influences that are not purely economic; it is easy enough in a dynamic context to specify an underlying and exogenously determined rate of migration, but ideally this will require the development of demographic accounts of the kind advocated by Stone.

[31] Thus the user can limit the migration possibilities to rural-urban pathways and/or other pathways, including urban-rural.

$$YMIGR_{ins,insp} = \frac{\left[\sum_{f} INSVA_{insp,f}\right]}{\left[\sum_{f} INSVA_{ins,f}\right]} \qquad \forall map\_insw\_inswp_{ins,insp} \tag{C4}$$

$$YMIGR.FX_{ins,insp} = YMIGRA_{ins,insp} \qquad \not\forall map\_insw\_inswp_{ins,insp} \tag{C4b}$$

$$YMIGR.FX_{ins,ins} = 0.0 \tag{C4c}$$

$$FSIM\_I_{f,ins,insp} = fsia_{ins,f} * \left(\frac{YMIGR_{ins,insp}}{YMIGRA_{ins,insp}}\right)^{etamig_{f,\text{ins},inswp}} - fsia_{ins,f}\ . \qquad \forall map\_insw\_inswp_{ins,insp} \tag{C5}$$

$$FSIM\_I_{f,ins,insp} = fsia_{ins,f} - \sum_{insp\$\left(notsameas_{ins,insp}\right)} FSIM\_I_{f,ins,insp} \qquad \forall inswmig_{ins} \textbf{ AND } fsia_{ins,f} \tag{C6}$$

$$FSIM\_I.FX_{ins,insp} = 0.0 \qquad \not\forall map\_insw\_inswp_{ins,insp} \tag{C6b}$$

$$FSIM\_I.LO_{ins,ins} = -\inf \tag{C6c}$$

$$FSIM\_I.LO_{ins,ins} = +\inf \tag{C6d}$$

$$FSIM\_I.L_{f,ins,ins} = fsia_{ins,f} \tag{C6e}$$

$$FSIM\_I.FX_{f,ins,insp} = 0.0 \qquad \not\forall fsia_{ins,f} = 0.0 \tag{C6f}$$

$$FSI\_I_{ins,f} = \sum_{insp\$inswmig_{insp}} FSIM\_I_{f,insp,ins} \qquad \not\forall inswmigp_{ins} \text{ and } FSI\_IA_{ins,f} \tag{C7}$$

$$FSI\_I.FX_{ins,f} = fsia_{ins,f} \qquad \not\forall inswmigp_{ins} \text{ and } FSI\_IA_{ins,f} \tag{C7b}$$

$$FSI\_I.FX_{ins,f} = 0.0 \qquad \forall fsia_{ins,f} = 0.0 \tag{C7c}$$

Given these changes in relative incomes and elasticities of migration (*etamig*), which are factor and institution pair specific, the quantities of households moving between RHG pairs for each institution (*FSIM_I*) can be determined (C5) by a constant elasticity of supply function and their summation across institutions produces the supply of each factor by each institution (*FSI*) in Eqn (C6). There is also a need to ensure that no additional households are created; this is achieved by a constraint equation that ensures that for each household moved from one segment only one household is created in the paired segment (C7).

The endogeneity of the functional distribution is ensured by the fact that the functional distribution of income depends upon the shares of factors supplied by different institutions (*FSISH*), which is a function of the supply of factors by institutions (*FSI*), which is defined in Eqn (C6). Since both of these are endogenous variables the functional distribution of income is endogenous.

An important behavioural assumption in STAGE_DEV is that RHGs are assumed to make the migration decision before making the factor mobility decision. Hence in the migration equation (C5) the constant supply elasticity functions operates on the base level of the factor supplies by the relevant institution/RHG (*FSIA*). This is in fact defined as *fsia*, which is equal to *FSI*0 in a comparative static context, but provides the ability to update *fsia* in the update statements between solution periods in a recursive dynamic setting.

Factor Mobility

A typical approach to factor supply and demand is that factors are rigidly segmented so that the total available supply of a factor is fixed, either at the current level of total demand, i.e., full employment, or at some level that is greater than the current level of total demand so as to allow for unemployment of that type of factor. This presumption of rigid segmentation is restrictive and as such does not allow for the possibility that labour can transition, to a greater or lesser extent, between the segments in response to changes in the factor market with or without job specific training. This can be especially problematic when labour types are classified by occupational categories[32], e.g., clerks, shop workers, agricultural workers, etc. For instance, **tractor** operators, in agriculture, may readily transition into **JCB** operators, in

[32] ILO data record labour by definitions based on occupational categories. Such a system of classification can provide useful descriptive statistics about the **current** patterns of employment. But CGE model simulations are overwhelmingly concerned with **changes** in patterns of employment in which case classification by occupational categories may be limiting. However, those compiling databases may be limited in the choice of classifications by the availability of data.

construction, but if they are classified as agricultural workers and construction workers respectively, many model close off this transition option. Moreover, a common option of classifying labour by levels of skill, e.g., skilled, semi-skilled and unskilled, involves the implicit assumption that all labour of a specific type receives the same average wage rate when within each type of labour there is likely to be a range of wage rates about the average. Thus it may be that lower paid skilled workers may be willing to take employment as semi-skilled workers if average wage rates for skilled workers decline relative to those of semi-skilled workers.

These differences in average wage rates are reflected in the fact that workers of the same notional type are typically paid at different wage rates according to the activity in which they are employed. This is evident in any CGE model for which there are data for both the transactions values and quantities of labour employed by different activities; in this model these differences are reported by the values of $WFDIST_{f,a}$. Where such data are available it is not uncommon to find that skilled agricultural workers are paid less than semi-skilled manufacturing workers. Such an observation implies, within the logic of CGE models and the labour classification scheme, that the labour market is not operating efficiently. Such an observation is typically, if at all, justified by some combination of assuming that there are non-rewarded preferences that explain the differences in average wages and/or that the differences are entirely due to activity specific attributes, i.e., the maintained assumption is that the labour classification scheme encompasses all differences in characteristics of the labour types. But it does mean that any reallocations of labour in simulations can and, typically, does result in changes in the average productivity of labour, which is equivalent to changing the factor endowments in the model.

The inclusion of the assumption of factor mobility across types of labour relaxes this restrictive assumption by assuming that types of factors can transition into other, specified, types in response to changes in the relative rates of return to factors (*WMIGR*); these are endogenous changes that are computed in Eqn (C8). Given these changes in relative rates of return and elasticities of mobility (*etaff*), which are factor pair and institution specific, the quantities of factors moving between factor pairs for each institution (*FSIM_F*) can be determined (C9) and their summation across institutions produces the supply of each factor by each institution (*FSI*) in Eqn (C10). There also a need to ensure that no additional factors are created; this achieved by a constraint equation that ensures that for each unit of a factor moved from one segment only one unit of a factor is created in the paired segment (C11).

Factor Mobility Block Equations

$$WMIGR_{f,fp,ins} = \frac{\left[\left(\sum_{a} WF_{fp} * WFDIST_{fp,a} * FD_{fp,a}\right) \Big/ \sum_{a} FD_{fp,a}\right]}{\left[\left(\sum_{a} WF_{f} * WFDIST_{f,a} * FD_{f,a}\right) \Big/ \sum_{a} FD_{f,a}\right]} \quad \text{(C8)}$$

$$\forall \left(map\_fmig\_fmigp_{f,fp} \textbf{ AND } FSI\_I0_{ins,f} \textbf{ AND } FSI\_I0_{ins,fp} \right)$$

$$WMIGR_{f,fp,ins} = WMIGRA_{f,fp,ins}$$

$$\not\forall \left(map\_fmig\_fmigp_{f,fp} \textbf{ AND } FSI\_I0_{ins,f} \textbf{ AND } FSI\_I0_{ins,fp} \right) \quad \text{(C8b)}$$

$$WMIGR_{f,f,ins} = 0.0 \quad \text{(C8c)}$$

$$FSIM\_F_{f,fp,ins} = FSI\_I_{ins,f} * \left(\frac{WMIGR_{f,fp,ins}}{WMIGRA_{f,fp,ins}} \right)^{etaff_{f,fp,ins}} - FSI\_I_{ins,f} \quad \text{(C9)}$$

$$\forall \left(map\_fmig\_fmigp_{f,fp} \textbf{ AND } FSI\_I0_{ins,f} \textbf{ AND } FSI\_I_{ins,fp} \right)$$

$$FSIM\_F_{f,f,ins} = FSI\_I_{ins,f} - \sum_{fp\$\left(notsameas_{fp,f}\right)} FSIM\_F_{f,fp,ins} \quad \text{(C10)}$$

$$\forall fmig_{f} \textbf{ AND } FSI\_I0_{ins,f}$$

$$FSIM\_F.FX_{f,fp,ins} = 0.0$$

$$\not\forall \left(fmig_{f} \textbf{ AND } FSI\_I0_{ins,f} \textbf{ AND } FSI\_I0_{ins,fp} \right) \quad \text{(C10b)}$$

$$FSIM\_F.LO_{f,f,ins} = -\inf \quad \text{(C10c)}$$

$$FSIM\_F.UP_{f,f,ins} = +\inf \quad \text{(C10d)}$$

$$FSIM\_F.L_{f,f,ins} = fsia_{ins,f} \quad \text{(C10e)}$$

$$FSIM\_F.FX_{f,f,ins} = 0.0 \qquad \forall FSI0_{ins,f} = 0.0 \quad \text{(C10f)}$$

$$FSI_{ins,f} = \sum_{fp\$\,fmig_{fp}} FSIM\_F_{fp,f,ins} \qquad \forall fmig_{f} \textbf{ AND } FSI0_{ins,f} \quad \text{(C11)}$$

Note however that the mobility of factors is defined in (C9) by reference to the numbers of factors supplied by each institution AFTER household migration, i.e., *FSI_I*, and not by reference to the amounts of factors supplied in the base period (*fsia*). This reflects the implicit presumption that household migration takes place before factor mobility decisions and ensures that in the solution the variables are all endogenous.[33] Similarly equation (C10) is also based on the post migration factor supplies *FSIS_I*, rather than the base quantities, *FSIM_F*.

The endogeneity of the functional distribution is ensured by the fact that the functional distribution of income depends upon the shares of factors supplied by different institutions (*FSISH*) that is a function of the supply of factors by institutions (*FSI*). Since both of these are endogenous variables the functional distribution of income is endogenous.

Commodity Market Clearing

Market clearing for the composite commodity markets requires that the supplies of the composite commodity (*QQ*) are equal to total of domestic demands for composite commodities, which consists of intermediate demand (*QINTD*), household (*QCD* and *QCD*2), enterprise (*QED*) and government (*QGD*) and investment (*QINVD*) final demands (C13). Note how the market clearing condition with respect to final demand by households has to be formulated so as to avoid double counting by ensuring that no aggregate commodities enter into the definition (RHS) of domestic demand. Since the markets for domestically produced commodities are also cleared (X16) this ensures a full clearing of all commodity markets. Similarly, it is necessary to ensure clearing of the production of differentiated commodities by activities when activities can adjust their output mixes in response to changes in relative commodity prices; this is done in equation (C12).

Commodity Market Clearing Block Equations

$$QXAC_{a,c} = IOQXACQX_{a,c} + QX_{a} \tag{C12}$$

33 Since the solutions are simultaneous this is not critical but it does simplify exposition and model clarity.

$$QQ_c = QTTD_c + QINTD_c + \sum_h QCD_{c,h\$ccesn_c} + \sum_{cag}\sum_h QCD2_{cag,c,h\$cces_c} + \sum_e QED_{c,e} + QGD_c + \sum_i QINVD_{i,c} \quad \text{(C13)}$$

Macroeconomic Closure Block

Making savings a residual for each account clears the two institutional accounts that are not cleared elsewhere – government and rest of the world. Thus the government account clears (C14) by defining government savings (*KAPGOV*) as the difference between government income and other expenditures, i.e., a residual. The rest of world account clears (C15) by defining the balance on the capital account (*CAPWOR*) as the difference between expenditure on imports, of commodities and factor services, and total income from the rest of the world, which includes export revenues and payments for factor services, transfers from the rest of the world to the household, enterprise and government accounts, i.e., it is a residual.

Macroeconomic Closure Block Equations

$$KAPGOV = YG - EG\,. \quad \text{(C14)}$$

$$CAPWOR = \left(\sum_c PWM_c * QM_c\right) + \left(\sum_f \frac{YFWOR_f}{ER}\right) - \left(\sum_c PWE_c * QE_c\right) - \left(\sum_f factwor_f\right) - \left(\sum_h howor_h\right) - entwor - govwor \quad . \quad \text{(C15)}$$

Absorption Closure

The total value of domestic final demand (*VFDOMD*) is defined (C16) as the sum of the expenditures on final demands by households and other domestic institutions (enterprises, government and investment). Note again that the value of final demand must exclude the demand for aggregate commodities to avoid double counting.

It is also useful to express the values of final demand by each non-household domestic institution as a proportion of the total value of domestic final demand; this allows the implementation of what has been called a ‘balanced macroeconomic closure’.[34] Hence the share of the value of final demand by enterprises (C17) can be defined as a proportion of total final domestic demand, and similarly for government’s value share of final demand (C18) and for investment’s value share of final demand (C19).

If the share variables (*VEDSH*, *VGDSH* and *INVESTSH*) are fixed then the quantity adjustment variables on the associated volumes of final demand by domestic non-household institutions (*QEDADJ*, *QGDADJ* and *IADJ* or *S*ADJ*) must be free to vary. On the other hand if the volume adjusters are fixed the associated share variables must be free so as to allow the value of final demand by ‘each’ institution to vary.

Absorption Closure Block Equations

$$\begin{aligned} VFDOMD = & \left(\sum_{c,h} PQD_c * \left(1+TV_c\right) * QCD_{c,h\$ccesn_c} \right) \\ & + \left(\sum_{cag,h} PQD_c * \left(1+TV_c\right) * QCD2_{cag,c,h\$cces_c} \right) \\ & + \left(\sum_{e,c} PQD_c * QED_{e,c} \right) + \left(\sum_{c} PQD_c * QGD_c \right) + \left(\sum_{i,c} PQD_c * QINVD_{i,c} \right) \end{aligned} \tag{C16}$$

$$VEDSH_e = VED_e / VFDOMD \tag{C17}$$

$$VGDSH = VGD / VFDOMD \tag{C18}$$

$$INVESTSH = INVEST / VFDOMD . \tag{C19}$$

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34 The adoption of such a closure rule for this class of model has been advocated by Sherman Robinson and is a feature, albeit implemented slightly differently, of the IFPRI standard model.

Slack

The final account to be cleared is the capital account. Total savings (*TOTSAV*), see I3 above, is defined within the model and hence there has been an implicit presumption in the description that the total value of investment (*INVEST*) is driven by the volume of savings. This is the market clearing condition imposed by (C20). But this market clearing condition includes another term, *WALRAS*, which is a slack variable that returns a zero value when the model is fully closed and all markets are cleared, and hence its inclusion provides a quick check on model specification.

SLACK Block Equations

$$TOTSAV = INVEST + WALRAS \ . \qquad \text{(C20)}$$

GDP

It is not necessary to include a variable in the model for GDP, since GDP is a simple summary 'variable' that can be calculated from the simulation results. However, it is convenient in some circumstances, e.g., while benchmarking a recursive dynamic model, to include GDP as a variable. In this model GDP is included as a variable that is calculated from the expenditure side, i.e., domestic absorption (valued a purchaser prices)[35] plus exports (valued at basic prices) less imports (valued at basic prices), (C21).

GDP Block Equations

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[35] Again note the need to avoid double counting that would occur if aggregate commodities were included.

$$
\begin{aligned}
GDP = &\left(\sum_{c,h} PQD_c * \left(1+TV_c\right) * QCD_{c,h\$ccesn_c}\right) \\
&+\left(\sum_{cag,h} PQD_c * \left(1+TV_c\right) * QCD2_{cag,c,h\$cces_c}\right) \\
&+\left(\sum_{e,c} PQD_c * QED_{e,c}\right) + \left(\sum_{c} PQD_c * QGD_c\right) + \left(\sum_{i,c} PQD_c * QINVD_{i,c}\right) \\
&+\left(\sum_{c} PWE_c * QE_c * ER\right) - \left(\sum_{c} PWM_c * QM_c * ER\right)
\end{aligned} \tag{C21}
$$

Model Closure Conditions or Rules

In mathematical programming terms the model closure conditions are, at their simplest, a matter of ensuring that the numbers of equations and variables are consistent. However economic theoretic dimensions of model closure rules are more complex, and, as would be expected in the context of an economic model, more important. The essence of model closure rules is that they define important and fundamental differences in perceptions of how an economic system operates (see Sen, 1963; Pyatt, 1987; Kilkenny and Robinson, 1990). The closure rules can be perceived as operating on two levels; on a general level whereby the closure rules relate to macroeconomic considerations, e.g., is investment expenditure determined by the volume of savings or exogenously, and on a specific level where the closure rules are used to capture particular features of an economic system, e.g., the degree of intersectoral capital mobility.

This model allows for a range of both general and specific closure rules. The discussion below provides details of the main options available with this formulation of the model by reference to the accounts to which the rules refer.

<u>Foreign Exchange Account Closure</u>

The closure of the rest of the world account can be achieved by fixing either the exchange rate variable (AC1a) or the balance on the current account (AC1b). Fixing the exchange rate is appropriate for countries with a fixed exchange rate regime whilst fixing the current account balance is appropriate for countries that face restrictions on the value of the current account balance, e.g., countries following structural adjustment programmes. It is a common practice to fix a variable at its initial level by using the associated parameter, i.e., ***0, but it is possible to fix the variable to any appropriate value.

The model is formulated with the world prices for traded commodities declared as variables, i.e., PWM_c and PWE_c. If a strong small country assumption is adopted, i.e., the country is assumed to be a price taker on all world commodity markets, and then all world prices will be fixed. When calibrating the model the world prices will be fixed at their initial levels, (AC1c), but this does not mean they cannot be changed as parts of experiments.

However, the model allows a relaxation of the strong small country assumption, such that the country may face a downward sloping demand curve for one or more of its export commodities. Hence the world prices of some commodities are determined by the interaction of demand and supply on the world market, i.e., they are variables. This is achieved by limiting the range of world export prices that are fixed to those for which there are no export demand function, (AC1d), by selecting membership of the set *cedn*.

Foreign Exchange Market Closure Equations

$$ER = \overline{ER} \qquad \text{(AC1a)}$$

$$CAPWOR = \overline{CAPWOR} \qquad \text{(AC1b)}$$

$$\begin{aligned} PWE_c &= \overline{PWE_c} \\ PWM_c &= \overline{PWM_c} \end{aligned} \qquad \text{(AC1c)}$$

$$PWE_{cedn} = \overline{PWE_{cedn}} \qquad \text{(AC1d)}$$

Capital Account Closure

To ensure that aggregate savings equal aggregate investment, the determinants of either savings or investment must be fixed. There are multiple ways of achieving this result. For instance this can be achieved by fixing either the saving rates for households or the volumes of commodity investment. This involves fixing either the savings rates adjusters (AC2a) or the investment volume adjuster (AC2c). Note that fixing the investment volume adjuster (AC2b) means that the value of investment expenditure might change due to changes in the prices of investment commodities (*PQD*). Note also that only one of the savings rate adjusters should be fixed; if *SADJ* is fixed the adjustment in such cases takes place through equiproportionate changes in the savings rates of households and enterprises, if *SHADJ* is fixed the adjustment in such cases takes place through equiproportionate changes in the savings rates of households, and if *SEADJ* is fixed the adjustment in such cases takes place through equiproportionate changes in the savings rates of enterprises. Alternatively savings rates can be adjusted through the additive adjustment factors (DS, DSHH, DSEN) with the same relationships between the savings rates of different classes of institutions (AC2b). Note that there are other sources of savings. The magnitudes of these other savings sources can also be changed through the closure rules (see below).

Capital Account Closure Equations

$$\begin{aligned} SADJ &= \overline{SADJ} \\ SHADJ &= \overline{SHADJ} \\ SEADJ &= \overline{SEADJ} \end{aligned} \qquad \text{(AC2a)}$$

$$\begin{aligned} DS &= \overline{DS} \\ DSHH &= \overline{DSHH} \\ DSEN &= \overline{DSEN} \end{aligned} \qquad \text{(AC2b)}$$

$$IADJ = \overline{IADJ} \qquad \text{(AC2c)}$$

$$INVEST = \overline{INVEST} \qquad \text{(AC2d)}$$

$$INVESTSH = \overline{INVESTSH} \qquad \text{(AC2e)}$$

Fixing savings, and thus deeming the economy to be savings-driven, could be considered a Neo-Classical approach. Closing the economy by fixing investment could be construed as making the model reflect the Keynesian investment-driven assumption for the operation of an economy.

The model includes a variable for the value of investment (*INVEST*), which can also be used to close the capital account (AC2d). If *INVEST* is fixed in an investment driven closure, then the model will need to adjust the savings rates to maintain equilibrium between the value of savings (*TOTSAV*) and the fixed value of investment. This can only be achieved by changes in the volumes of commodities demanded for investment (*QINVD*) or their prices (*PQD*). But the prices (*PQD*) depend on much more than investment, hence the main adjustment must take place through the volumes of commodities demanded, i.e., *QINVD*, and therefore the volume adjuster (*IADJ*) must be variable, as must the savings rate adjuster (*SADJ*).

Alternatively the share of investment expenditure in the total value of domestic final demand can be fixed, (AC2e), which means that the total value of investment is fixed by reference to the value of total final demand, but otherwise the adjustment mechanisms follow the same processes as for fixing *INVEST* equal to some level.

Enterprise Account Closure

Fixing the volumes of commodities demand by enterprises, (AC3a), closes the enterprise account. Note that this rule allows the value of commodity expenditures by the enterprise account to vary, which *ceteris paribus* means that the value of savings by enterprises (*CAPENT*) and thus total savings (*TOTSAV*) vary. If the value of this adjuster is changed, but left fixed, this imposes equiproportionate changes on the volumes of commodities demanded.

Enterprise Account Closure Equations

$$QEDADJ = \overline{QEDADJ} \qquad \text{(AC3a)}$$

$$VED = \overline{VED}\,. \qquad \text{(AC3b)}$$

$$VEDSH = \overline{VEDSH} \qquad \text{(AC3c)}$$

$$HEADJ = \overline{HEADJ} \qquad \text{(AC3d)}$$

If *QEDADJ* is allowed to vary then another variable must be fixed; the most likely alternative is the value of consumption expenditures by enterprises (*VED*) (AC3b). This would impose adjustments through equiproportionate changes in the volumes of commodities demanded, and would feed through so that enterprise savings (*CAPENT*) reflecting directly the changes in the income of enterprises (*YE*). Alternatively the share of enterprise expenditure in the total value of domestic final demand can be fixed, (AC3c), which means that the total value of enterprise consumption expenditure (*VED*) is fixed by reference to the value of total final demand, but otherwise the adjustment mechanisms follow the same processes as for fixing *VED* equal to some level.

Finally the scaling factor for enterprise transfers to households (*HEADJ*) needs fixing (AC3d).

Government Account Closure

The closure rules for the government account are slightly more tricky because they are important components of the model that are used to investigate fiscal policy considerations. The base specification uses the assumption that government savings are a residual; when the determinants of government income and expenditure are ‘fixed’, government savings must be free to adjust.

Thus in the base specification all the tax rates (variables) are fixed by declaring the base tax rates as parameters and then fixing all the multiplicative and additive tax rate scaling factors (AC4a – AC4r).

Tax Rate Adjustment Closure Equations

$$TMADJ = \overline{TMADJ} \quad \text{(AC4a)}$$

$$TEADJ = \overline{TEADJ} \quad \text{(AC4b)}$$

$$TSADJ = \overline{TSADJ} \quad \text{(AC4c)}$$

$$TQSADJ = \overline{TQSADJ} \quad \text{(AC4d)}$$

$$TVADJ = \overline{TVADJ} \quad \text{(AC4e)}$$

$$TEXADJ = \overline{TEXADJ} \quad \text{(AC4f)}$$

$$TXADJ = \overline{TXADJ} \quad \text{(AC4g)}$$

$$TFADJ = \overline{TFADJ} \quad \text{(AC4h)}$$

$$TYADJ = \overline{TYADJ} \quad \text{(AC4i)}$$

$$TYEADJ = \overline{TYEADJ} \quad \text{(AC4j)}$$

$$TYHADJ = \overline{TYHADJ} \quad \text{(AC4k)}$$

$$DTM = \overline{DTM} \quad \text{(AC4l)}$$

$$DTE = \overline{DTE} \quad \text{(AC4m)}$$

$$DTS = \overline{DTS} \quad \text{(AC4n)}$$

$$DTQS = \overline{DTQS} \quad \text{(AC4o)}$$

$$DTV = \overline{DTV} \quad \text{(AC4p)}$$

$$DTEX = \overline{DTEX} \quad \text{(AC4q)}$$

$$DTX = \overline{DTX} \quad \text{(AC4r)}$$

$$DTF = \overline{DTF} \quad \text{(AC4s)}$$

$$DTYF = \overline{DTYF} \quad \text{(AC4t)}$$

$$DTYH = \overline{DTYH} \quad \text{(AC4u)}$$

$$DTYE = \overline{DTYE}\,. \quad \text{(AC4v)}$$

$$MTAX = \overline{MTAX}\,. \quad \text{(AC4v)}$$

$$ETAX = \overline{ETAX}\ . \qquad \text{(AC4v)}$$

$$STAX = \overline{STAX}\ . \qquad \text{(AC4v)}$$

$$QSTAX = \overline{QSTAX}\ . \qquad \text{(AC4v)}$$

$$VTAX = \overline{VTAX}\ . \qquad \text{(AC4v)}$$

$$EXTAX = \overline{EXTAX}\ . \qquad \text{(AC4v)}$$

$$FTAX = \overline{FTAX}\ . \qquad \text{(AC4v)}$$

$$ITAX = \overline{ITAX}\ . \qquad \text{(AC4v)}$$

$$FYTAX = \overline{FYTAX}\ . \qquad \text{(AC4v)}$$

$$DTAX = \overline{DTAX}\ . \qquad \text{(AC4v)}$$

Consequently changes in tax revenue to the government are consequences of changes in the other variables that enter into the tax income equations (GR1 to GR8). The two other sources of income to the government are controlled by parameters, *govvash* and *govwor*, and therefore are not a source of concern for model closure.[36]

Also note that because there are equations for the revenues by each tax instrument (GR1 to GR8) it is straightforward to adjust the tax rates to achieve a given volume of revenue from each tax instrument; this type of arrangement is potentially useful in circumstances where is is argued/believed that binding constraints upon the revenue possibilities from specific tax instruments.

In the base specification government expenditure is controlled by fixing the volumes of commodity demand (*QGD*) through the government demand adjuster (*QGDADJ*) in (AC4s). Alternatively either the value of government consumption expenditure (*VGD*) can be fixed, (AC4t), or the share of government expenditure in the total value of domestic final demand

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36 The values of income from non-tax sources can of course vary because each component involves a variable.

(*VGDSH*) can be fixed, (AC4u). The scaling factor on the values of transfers to households and enterprises through the household (*HGADJ*) and enterprise (*EGADJ*) adjusters, (AC4v and AC4w) also need to be fixed.

Government Expenditure Closure Equations

$$QGDADJ = \overline{QGDADJ} \quad \text{(AC4s)}$$

$$VGD = \overline{VGD} \quad \text{(AC4t)}$$

$$VGDSH = \overline{VGDSH}\,. \quad \text{(AC4u)}$$

$$HGADJ = \overline{HGADJ} \quad \text{(AC4v)}$$

$$EGADJ = \overline{EGADJ} \quad \text{(AC4w)}$$

$$CAPGOV = \overline{CAPGOV} \quad \text{(AC4x)}$$

This specification ensures that all the parameters that the government can/does control are fixed and consequently that the only determinants of government income and expenditure that are free to vary are those that the government does not directly control. Hence the equilibrating condition is that government savings, the internal balance, is not fixed.

If however the model requires government savings to be fixed (AC4x), then either government income or expenditure must be free to adjust. Such a condition might reasonably be expected in many circumstances, e.g., the government might define an acceptable level of borrowing or such a condition might be imposed externally.

In its simplest form this can be achieved by allowing one of the previously fixed adjusters (AC4a to AC4w) to vary. Thus if the sales tax adjuster (*TSADJ*) is made variable then the sales tax rates will be varied equiproportionately so as to satisfy the internal balance condition. More complex experiments might result from the imposition of multiple conditions, e.g., a halving of import duty rates coupled with a reduction in government deficit, in which case the variables *TMADJ* and *KAPGOV* would also require resetting. But these conditions might create a model that is infeasible, e.g., due to insufficient flexibility through the sales tax mechanism, or unrealistically high rates of sales taxes. In such circumstances it may be necessary to allow adjustments in multiple tax adjusters. One method then would be to fix the tax adjusters to move in parallel with each other.

However, if the adjustments only take place through the tax rate scaling factors the relative tax rates will be fixed. To change relative tax rates it is necessary to change the relevant tax parameters. Typically such changes would be implemented in policy experiment files rather than within the closure section of the model.

Numéraire

The model specification allows for a choice of two price normalisation equations (AC5a and AC5b), the consumer price index (CPI) and a producer price index (PPI). A *numéraire* is needed to serve as a base since the model is homogenous of degree zero in prices and hence only defines relative prices.

Numéraire Closure Equations

$$CPI = \overline{CPI} \qquad \text{(AC5a)}$$

$$PPI = \overline{PPI}\,. \qquad \text{(AC5b)}$$

Factor Market Closure

The factor market closure rules are more difficult to implement than many of the other closure rules. Hence the discussion below proceeds in three stages; the first stage sets up a basic specification whereby all factors are deemed perfectly mobile, the second stage introduces a more general specification whereby factors can be made activity specific and allowance can be made for unemployed factors, while the third stage introduces the idea that factor market restrictions may arise from activity specific characteristics, rather than the factor inspired restrictions considered in the second stage.

Full Factor Mobility and Employment Closure

This factor market closure requires that the total supply (*FSI*) of and total demand for factors (*FD*) equate (AC6a). The total supplies of each factor are determined exogenously and hence defines the first set of factor market closure conditions. The demands for factor *f* by activity *a* and the wage rates for factors are determined endogenously. But the model specification includes the assumption that the wage rates for factors are averages, by allowing for the possibility that the payments to notionally identical factors might vary across activities through the variable that captures the 'sectoral proportions for factor prices'. These proportions are assumed to be a consequence of the use made by activities of factors, rather than of the factors themselves, and are therefore assumed fixed, (AC6b). Finally while it may seem that factor prices must be limited to positive values the actual bounds placed upon the average factor prices, (AC6c) are plus or minus infinity. This is a consequence of the use of the PATH solver.

Basic Factor Market Closure Equations

$$\sum_{insw} FSI_{insw,f} = \overline{FS}_f \qquad \text{(AC6a)}$$

$$WFDIST_{f,a} = \overline{WFDIST_{f,a}} \;. \qquad \text{(AC6b)}$$

$$\begin{array}{l} \text{Min } WF_f = -\text{infinity} \\ \text{Max } WF_f = +\text{infinity} \end{array} \qquad \text{(AC6c)}$$

Factor Immobility and/or Unemployment Closures

More general factor market closures wherein factor immobility and/or factor unemployment are assumed can be achieved by determining which of the variables referring to factors are treated as variables and which of the variables are treated as factors. If factor market closure rules are changed it is important to be careful to preserve the equation and variable counts when relaxing conditions, i.e., converting parameters into variables, and imposing conditions, i.e., converting variables into parameters, while preserving the economic logic of the model.

A convenient way to proceed is to define a block of conditions for each factor. For this model this amounts to defining the following possible equations (AC6d) where *fact* indicates the specific factor and *activ* a specific activity. This block of equations includes all the variables that were declared for the model with reference to factors plus an extra equation for *WFDIST*, i.e., $WFDIST_{fact,activ} = \overline{WFDIST}_{fact,activ}$, whose role will be defined below. The choice of which equations are binding and which are not imposed will determine the factor market closure conditions.

Factor Block Equations

$$\begin{aligned} FS_{fact} &= \overline{FS}_{fact} \\ WFDIST_{fact,a} &= \overline{WFDIST}_{fact,a} \\ \text{Min } WF_{fact} &= -\text{infinity} \\ \text{Max } WF_{fact} &= +\text{infinity} \\ FD_{fact,a} &= \overline{FD}_{fact,a} \\ WF_{fact} &= \overline{WF}_{fact} \\ WFDIST_{fact,activ} &= \overline{WFDIST}_{fact,activ} \\ \text{Min } FS_{fact} &= -\text{infinity} \\ \text{Max } FS_{fact} &= +\text{infinity} \end{aligned} \qquad \text{(AC6d)}$$

As can be seen the first four equations in the block (AC6d) are the same as those in the 'Full Factor Mobility and Employment Closure'; hence ensuring that these four equations are operating for each of the factors is a longhand method for imposing the 'Full Factor Mobility and Employment Closure'. Assume that this set of conditions represents a starting point, i.e., the first four equations are binding and the last five equations are not imposed.

Assume now that it is planned to impose a short run closure on the model, whereby a factor is assumed to be activity specific, and hence there is no inter sectoral factor mobility. Typically this would involve making capital activity specific and immobile, although it can be applied to any factor. This requires imposing the condition that factor demands are activity specific, i.e., the condition ($FD_{fact,a} = \overline{FD}_{fact,a}$) must be imposed. But the returns to this factor in different uses (activities) must now be allowed to vary, i.e., the condition (AC6b) must now be relaxed.

The number of imposed conditions is equal to the number of relaxed conditions, which suggests that the model will still be consistent. But the condition fixing the total supply of the factor is redundant since if factor demands are fixed the total factor supply cannot vary. Hence the condition (AC6a) is redundant and must be relaxed. Hence at least one other condition must be imposed to restore balance between the numbers of equations and variables. This can be achieved by fixing one of the sectoral proportions for factor prices for a specific activity, i.e., (AC6b), which means that the activity specific returns to the factor will be defined relative to the return to the factor in *activ*.[37]

Factor Market Closure Equations

$$FD_{fact,a} = \overline{FD_{fact,a}} \qquad \text{(AC6e)}$$

$$WFDIST_{fact,a} = \overline{WFDIST_{fact,a}} \qquad \text{(AC6f)}$$

$$FS_{fact} = \overline{FS_{fact}} \qquad \text{(AC6g)}$$

$$WFDIST_{fact,activ} = \overline{WFDIST_{fact,activ}} \qquad \text{(AC6h)}$$

$$WF_{fact} = \overline{WF_{fact}} \qquad \text{(AC6i)}$$

$$FS_{fact} = \overline{FS_{fact}} \qquad \text{(AC6j)}$$

$$\begin{matrix} \text{Min } FS_{fact} = 0 \\ \text{Max } FS_{fact} = +\text{infinity} \end{matrix} \qquad \text{(AC6k)}$$

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[37] It can be important to ensure a sensible choice of reference activity. In particular this is important if a factor is not used, or little used, by the chosen activity.

Start again from the closure conditions for full factor mobility and employments and then assume that there is unemployment of one or more factors in the economy; typically this would be one type or another of unskilled labour. If the supply of the unemployed factor is perfectly elastic, then activities can employ any amount of that factor at a fixed price. This requires imposing the condition that factor prices are fixed (AC6i) and relaxing the assumption that the total supply of the factor is fixed at the base level, i.e., relaxing (AC6a). It is useful however to impose some restrictions on the total supply of the factor that is unemployed. Hence the conditions (AC6k) can be imposed.[38]

Activity Inspired Restrictions on Factor Market Closures

There are circumstances where factor use by an activity might be restricted as a consequence of activity specific characteristics. For instance it might be assumed that the volume of production by an activity might be predetermined, e.g., known mineral resources might be fixed and/or there might be an exogenously fixed restriction upon the rate of extraction of a mineral commodity. In such cases the objective might be to fix the quantities of all factors used by an activity, rather than to fix the amounts of a factor used by all activities. This is clearly a variation on the factor market closure conditions for making a factor activity specific.

Factor Market Clearing Equations

$$FD_{f,activ} = \overline{FD_{f,activ}} \qquad \text{(AC6l)}$$

$$WFDIST_{f,activ} = \overline{WFDIST_{f,activ}} \qquad \text{(AC6m)}$$

If all factors used by an activity are fixed, this requires imposing the conditions that factor demands are fixed, (AC6l), where *activ* refers to the activity of concern. But the returns to these factors in this activities must now be allowed to vary, i.e., the conditions (AC6m) must now be relaxed. In this case the condition fixing the total supply of the factor is not

[38] If the total demand for the unemployed factor increases unrealistically in the policy simulations then it is possible to place an upper bound of the supply of the factor and then allow the wage rate from that factor to vary.

redundant since only the factor demands by *activ* are fixed and the factor supplies to be allocated across other activities are the total supplies unaccounted for by *activ*.

Such conditions can be imposed by extending the blocks of equations for each factor in the factor market closure section. However, it is often easier to manage the model by gathering together factor market conditions that are inspired by activity characteristics after the factor inspired equations. In this context it is useful to note that when working in GAMS that the last condition imposed, in terms of the order of the code, is binding and supersedes previous conditions.

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Appendix 1: STAGE Model Genalogy

The STAGE model started life in the mid-1990s. After initial (futile) struggles with the Cameroon CGE model then in the GAMS library Sherman Robinson gave Scott a copy of the single country CGE model developed for the US Department of Agriculture's (USDA) Economic Research Service (ERS) under Sherman's leadership (Robinson *et al*., 1990; Kilkenny, 1991). The USDA ERS model was based on an input-output representation of the inter industry transactions that limited the applicability of the model for the analyses of the decisions made by multi-product activities. This concern was raised with Sherman and Hans Lofgren in the late 1990s[39]; this problem was addressed by Hans and Sherman and a copy of the solution was shared with Scott. The developments by Hans and Sherman at IFPRI ultimately resulted in the production of the IFPRI standard model in 2001 (Lofgren *et al*., 2001). Consequently the IFPRI standard and STAGE models share a common heritage and a number of features although there also differences.

The USDA ERS model provided the basis for the PROVIDE project model (McDonald, 2003). This model also included the treatment of multi-product activities developed for the IFPRI Standard model but included different treatments of margins and differences that reflected issues relevant to South Africa at that time. Valuable contributions to the PROVIDE model were made by Cecilia Punt, Melt van Schoor, Lindsay Chant and Kalie Pauw. Melt van Schoor also made major contributions through the development of the SAMGator and SeeResults interfaces. An energy version of the PROVIDE model was developed with Jonah Tlhalefang and was used in Jonah's PhD thesis at the University of Sheffield. The 'final'/most developed version of the PROVIDE model appeared in Cecilia Punt's PhD thesis from the University of Stellenbosch, which among other things, included explicit modelling of changes in the composition of outputs by activities.

The PROVIDE project model developed into the STAGE model as part of the process of developing the GLOBE model from 2002 with Karen Thierfelder. Karen had also started her modelling career using the USDA ERS model and the NAFTA model in the 1990s.

The GLOBE model used a simplified variant of the STAGE model as the basis for the development of the within country/region behavioural equations. This process generated some

[39] The issue had become relevant when estimating the implications of BSE (McDonald and Roberts, 1998).

changes in behavioural relationship, code structure, methods for analyzing results and notation. Consequently in 2005 the STAGE 1 model was consolidated from previous models and made open source with some revisions from 2009.

The STAGE 2 model is a consolidation of model developments since 2009. It embodies contributions made by Karen Thierfelder, Cecilia Punt, Emanuele Ferrari, Dorothee Flaig and Emerta Aragie.

The STAGE model is part of a suite of models that include two global models (GLOBE and R23 models) and a range of teaching models – the SMOD suite. All these model use a (overwhelmingly) common set of notation and formats.

Appendix 2: Parameter, Variable and Equation Lists

STAGE_DEV

The parameter and variable listings are in alphabetic order, and are included for reference purposes. The parameters listed below are those used in the behavioural specifications/equations of the model, in addition to these parameters there are a further set of parameters. This extra set of parameters is used in model calibrated and for deriving results; there is one such parameter for each variable and they are identified by appending a '0' (zero) to the respective variable name.

Parameter List

Parameter Name	Parameter Description
ac(c)	Shift parameter for Armington CES function
actcomactsh(a,c)	Share of commodity c in output by activity a
actcomcomsh(a,c)	Share of activity a in output of commodity c
adva(a)	Shift parameter for CES production functions for QVA
adx(a)	Shift parameter for CES production functions for QX
adxc(c)	Shift parameter for commodity output CES aggregation
alphah(c,h)	Expenditure share by commodity c for household h
at(c)	Shift parameter for Armington CET function
beta(c,h)	Marginal budget shares
caphosh(h)	Shares of household income saved (after taxes)
comactactco(c,a)	intermediate input output coefficients
comactco(c,a)	use matrix coefficients
comentconst(c,e)	Enterprise demand volume
comgovconst(c)	Government demand volume
comhoav(c,h)	Household consumption shares
comtotsh(c)	Share of commodity c in total commodity demand
dabte(c)	Change in base export taxes on comm'y imported from region w
dabtex(c)	Change in base excise tax rate
dabtfue(c)	Change in base fuel tax rate
dabtm(c)	Change in base tariff rates on comm'y imported from region w
dabts(c)	Change in base sales tax rate
dabtx(a)	Change in base indirect tax rate
dabtye(e)	Change in base direct tax rate on enterprises
dabtyf(f)	Change in base direct tax rate on factors
dabtyh(h)	Change in base direct tax rate on households
delta(c)	Share parameter for Armington CES function
deltava(f,a)	Share parameters for CES production functions for QVA
deltax(a)	Share parameter for CES production functions for QX
deltaxc(a,c)	Share parameters for commodity output CES aggregation
deprec(f)	depreciation rate by factor f
dstocconst(c)	Stock change demand volume
econ(c)	constant for export demand equations
entgovconst(e)	Government transfers to enterprise e
entvash(e,f)	Share of income from factor f to enterprise e
entwor(e)	Transfers to enterprise e from world (constant in foreign currency)
eta(c)	export demand elasticity

Parameter Name	Parameter Description

factwor(f)	Factor payments from RoW (constant in foreign currency)
frisch(h)	Elasticity of the marginal utility of income
gamma(c)	Share parameter for Armington CET function
goventsh(e)	Share of entp' income after tax save and consump to govt
govvash(f)	Share of income from factor f to government
govwor	Transfers to government from world (constant in foreign currency)
hexps(h)	Subsistence consumption expenditure
hoentconst(h,e)	transfers to hhold h from enterprise e (nominal)
hoentsh(h,e)	Share of entp' income after tax save and consump to h'hold
hogovconst(h)	Transfers to hhold h from government (nominal but scalable)
hohoconst(h,hp)	interhousehold transfers
hohosh(h,hp)	Share of h'hold h after tax and saving income transferred to hp
hovash(h,f)	Share of income from factor f to household h
howor(h)	Transfers to household from world (constant in foreign currency)
invconst(c)	Investment demand volume
ioqintqx(a)	Agg intermed quantity per unit QX for Level 1 Leontief agg
ioqvaqx(a)	Agg value added quant per unit QX for Level 1 Leontief agg
kapentsh(e)	Average savings rate for enterprise e out of after tax income
predeltax(a)	dummy used to estimated deltax
pwse(c)	world price of export substitutes
qcdconst(c,h)	Volume of subsistence consumption
rhoc(c)	Elasticity parameter for Armington CES function
rhocva(a)	Elasticity parameter for CES production function for QVA
rhocx(a)	Elasticity parameter for CES production function for QX
rhocxc(c)	Elasticity parameter for commodity output CES aggregation
rhot(c)	Elasticity parameter for Output Armington CET function
sumelast(h)	Weighted sum of income elasticities
te01(c)	0-1 par for potential flexing of export taxes on comm'ies
tex01(c)	0-1 par for potential flexing of excise tax rates
tfue01(c)	0-1 par for potential flexing of fuel tax rates
tm01(c)	0-1 par for potential flexing of Tariff rates on comm'ies
ts01(c)	0-1 par for potential flexing of sales tax rates
tx01(a)	0-1 par for potential flexing of indirect tax rates
tye01(e)	0-1 par for potential flexing of direct tax rates on e'rises
tyf01(f)	0-1 par for potential flexing of direct tax rates on factors
tyh01(h)	0-1 par for potential flexing of direct tax rates on h'holds
use(c,a)	use matrix transactions
vddtotsh(c)	Share of value of domestic output for the domestic market
worvash(f)	Share of income from factor f to RoW
yhelast(c,h)	(Normalised) household income elasticities

Variable List

Variable Name	Variable Description
CAPGOV	Government Savings
CAPWOR	Current account balance
CPI	Consumer price index
DTAX	Direct Income tax revenue
DTE	Partial Export tax rate scaling factor
DTEX	Partial Excise tax rate scaling factor
DTFUE	Partial Fuel tax rate scaling factor
DTM	Partial Tariff rate scaling factor
DTS	Partial Sales tax rate scaling factor
DTX	Partial Indirect tax rate scaling factor
DTYE	Partial direct tax on enterprise rate scaling factor
DTYF	Partial direct tax on factor rate scaling factor
DTYH	Partial direct tax on household rate scaling factor
EG	Expenditure by government
EGADJ	Transfers to enterprises by government Scaling Factor
ER	Exchange rate (domestic per world unit)
ETAX	Export tax revenue
EXTAX	Excise tax revenue
FD(f,a)	Demand for factor f by activity a
FS(f)	Supply of factor f
FUETAX	Fuel tax revenue
FYTAX	Factor Income tax revenue
GOVENT(e)	Government income from enterprise e
HEADJ	Scaling factor for enterprise transfers to households
HEXP(h)	Household consumption expenditure
HGADJ	Scaling factor for government transfers to households
HOENT(h,e)	Household Income from enterprise e
HOHO(h,hp)	Inter household transfer
IADJ	Investment scaling factor
INVEST	Total investment expenditure
INVESTSH	Value share of investment in total final domestic demand
ITAX	Indirect tax revenue
MTAX	Tariff revenue
PD(c)	Consumer price for domestic supply of commodity c
PE(c)	Domestic price of exports by activity a
PINT(a)	Price of aggregate intermediate input
PM(c)	Domestic price of competitive imports of commodity c
PPI	Producer (domestic) price index
PQD(c)	Purchaser price of composite commodity c
PQS(c)	Supply price of composite commodity c
PVA(a)	Value added price for activity a
PWE(c)	World price of exports in dollars
PWM(c)	World price of imports in dollars
PX(a)	Composite price of output by activity a
PXAC(a,c)	Activity commodity prices
PXC(c)	Producer price of composite domestic output

Variable Name	Variable Description
QCD(c,h)	Household consumption by commodity c
QD(c)	Domestic demand for commodity c
QE(c)	Domestic output exported by commodity c
QENTD(c,e)	Enterprise consumption by commodity c
QENTDADJ	Enterprise demand volume Scaling Factor
QGD(c)	Government consumption demand by commodity c
QGDADJ	Government consumption demand scaling factor
QINT(a)	Aggregate quantity of intermediates used by activity a
QINTD(c)	Demand for intermediate inputs by commodity
QINVD(c)	Investment demand by commodity c
QM(c)	Imports of commodity c
QQ(c)	Supply of composite commodity c
QVA(a)	Quantity of aggregate value added for level 1 production
QX(a)	Domestic production by activity a
QXAC(a,c)	Domestic commodity output by each activity
QXC(c)	Domestic production by commodity c
SADJ	Savings rate scaling factor for BOTH households and enterprises
SEADJ	Savings rate scaling factor for enterprises
SHADJ	Savings rate scaling factor for households
STAX	Sales tax revenue
TE(c)	Export taxes on exported comm'y c
TEADJ	Export subsidy Scaling Factor
TEX(c)	Excise tax rate
TEXADJ	Excise tax rate scaling factor
TFUE(c)	Fuel tax rate
TFUEADJ	Fuel tax rate scaling factor
TM(c)	Tariff rates on imported comm'y c
TMADJ	Tarrif rate Scaling Factor
TOTSAV	Total savings
TS(c)	Sales tax rate
TSADJ	Sales tax rate scaling factor
TX(a)	Indirect tax rate
TXADJ	Indirect Tax Scaling Factor
TYE(e)	Direct tax rate on enterprises
TYEADJ	Enterprise income tax Scaling Factor
TYF(f)	Direct tax rate on factor income
TYFADJ	Factor Tax Scaling Factor
TYH(h)	Direct tax rate on households
TYHADJ	Household Income Tax Scaling Factor
VENTD(e)	Value of enterprise e consumption expenditure
VENTDSH(e)	Value share of Ent consumption in total final domestic demand
VFDOMD	Value of final domestic demand
VGD	Value of Government consumption expenditure
VGDSH	Value share of Govt consumption in total final domestic demand
WALRAS	Slack variable for Walras's Law
WF(f)	Price of factor f
WFDIST(f,a)	Sectoral proportion for factor prices
YE(e)	Enterprise incomes
YF(f)	Income to factor f
YFDISP(f)	Factor income for distribution after depreciation
YFWOR(f)	Foreign factor income
YG	Government income
YH(h)	Income to household h

Equation Listing

Equation and Variable Counts for the Model

Name	Equation	Number of Equations	Variable	Number of Variables
	EXPORTS BLOCK			
$PEDEF_c$	$PE_c = PWE_c * ER * (1 - TE_c) - \sum_m (ioqttqe_{m,c} * PTT_m) \quad \forall ce$	ce	PE_c	ce
CET_c	$QXC_c = at_c * \left(\gamma_c * QE_c^{rhot_c} + (1 - \gamma_c) * QD_c^{rhot_c} \right)^{\frac{1}{rhot_c}} \quad \forall ce \textbf{ AND } cd$	c	QD_c	c
$ESUPPLY_a$	$\frac{QE_c}{QD_c} = \left[\frac{PE_c}{PD_c} * \frac{(1 - \gamma_c)}{\gamma_c} \right]^{\frac{1}{(rhot_c - 1)}} \quad \forall ce \textbf{ AND } cd$	c	QE_c	c
$EDEMAND_c$	$QE_c = econ_c * \left(\frac{PWE_c}{pwse_c} \right)^{-eta_c} \quad \forall ced$			
$CETALT_c$	$QXC_c = QD_c + QE_c \quad \forall (cen \textbf{ AND } cd) \textbf{ OR } (ce \textbf{ AND } cdn)$			

Name	Equation	Number of Equations	Variable	Number of Variables
	IMPORTS BLOCK			
$PMDEF_c$	$PM_c = PWM_c * ER * (1 + TM_c) \quad \forall cm$	cm	PM_c	cm
$ARMINGTON_c$	$QQ_c = ac_c \left(\delta_c QM_c^{-rhoc_c} + (1-\delta_c) QD_c^{-rhoc_c} \right)^{-\frac{1}{rhoc_c}} \quad \forall cm \text{ AND } cx$	c	QQ_c	c
$COSTMIN_c$	$\frac{QM_c}{QD_c} = \left[\frac{PD_c}{PM_c} * \frac{\delta_c}{(1-\delta_c)} \right]^{\frac{1}{(1+rhoc_c)}} \quad \forall cm \text{ AND } cx$	c	QM_c	c
$ARMALT$	$QQ_c = QD_c + QM_c \quad \forall (cmn \text{ AND } cx) \text{ OR } (cm \text{ AND } cxn)$			

Name	Equation	Number of Equations	Variable	Number of Variables
	TRADE AND TRANSPORT MARGINS BLOCK			
$PTTDEF_m$	$PTT_m = \sum_c ioqtddtt_{c,m} * PQD_c$	m	PTT_m	m
$QTTDEF_m$	$QTT_m = \sum_c \left(ioqttqq_{m,c} * QQ_c \right) + \sum_c \left(ioqttqe_{m,c} * QE_c \right)$	m	QTT_m	m
$QTTDEQ_c$	$QTTD_c = \sum_m ioqtddtt_{c,m} * QTT_m$	c	$QTTD_c$	c

Name	Equation	Number of Equations	Variable	Number of Variables
	COMMODITY PRICE BLOCK			
$PQDDEF_c$	$PQD_c = PQS_c * (1+TS_c + TEX_c) + TQS_c + \sum_m (ioqttqq_{m,c} * PTT_m)$	c	PQD_c	c
$PQSDEF_c$	$PQS_c = \frac{PD_c * QD_c + PM_c * QM_c}{QQ_c} \quad \forall cd$ **OR** cm	c	PQS_c	c
$PXCDEF_c$	$PXC_c = \frac{PD_c * QD_c + (PE_c * QE_c)\$ce_c}{QXC_c} \quad \forall cx$	cx	PXC_c	cx
$PQCDDEF_{cc,h}$	$PQCD_{cag,h} * QCD_{cag,h} = \sum_{cces\$map\_cag\_c_{cag,ccess}} PQD_{cces} * (1+TV_{cces}) * QCD2_{cag,cces,h}$	(cag,h)	$PQCD_{cc,h}$	$(cag*h)$
	NUMERAIRE BLOCK			
$CPIDEF$	$CPI = \sum_c comtotsh_c * (PQD_c + (1+TV_c))$	1	CPI	1
$PPIDEF$	$PPI = \sum_c vddtotsh_c * PD_c$	1	PPI	1

Name	Equation	Number of Equations	Variable	Number of Variables
	PRODUCTION BLOCK			
$PXDEF_a$	$PX_a = \sum_c IOQXACQX_{a,c} * PXC_c$	a	PX_a	a
$PVADEF_a$	$PX_a * (1 - TX_a) * QX_a = (PVA_a * QVA_a) + (PINT_a * QINT_a)$	a	PV_a	a
$PINTDEF_a$	$PINT_a = \sum_c \left(ioqtdqd_{c,a} * PQD \right)_c$	a	$PINT_a$	a
$ADXEQ_a$	$ADX_a = \left[\left(adxb_a + dabadx_a \right) * ADXADJ \right] + \left(DADX * adx01_a \right)$	a	ADX_a	a
$QXPRODFN_a$	$QX_a = AD_a^x \left(\delta_a^x QVA_a^{-rhoc_a^x} + \left(1 - \delta_a^x\right) QINT_a^{-rhoc_a^x} \right)^{-\frac{1}{rhoc_a^x}}$ $\forall aqx_a$	a	QX_a	a
$QXFOC_a$	$\frac{QVA_a}{QINT_a} = \left[\frac{PINT_a}{PVA_a} * \frac{\delta_a^x}{\left(1 - \delta_a^x\right)} \right]^{\frac{1}{\left(1 + rhoc_a^x\right)}}$ $\forall aqx_a$	a	$QINT_a$	a
	$QINT_a = ioqintqx_a * QX_a$ $\forall aqx_a$			
$QVADEF$	$QVA_a = ioqvaqx_a * QX_a$ $\forall aqxn_a$			
$QINTDEF$	$QINT_a = ioqintqx_a * QX_a$ $\forall aqx_a$			

Name	Equation	Number of Equations	Variable	Number of Variables
ADVAEQa	$ADVA_a = \left[\left(advab_a + dabadva_a\right) * ADVAADJ\right] + \left(DADVA * adva01_a\right)$	a	$ADVA_a$	a
QVAPRODFN$_a$	$QVA_a = AD_a^{va} * \left[\sum_{f \$ \delta_{f,a}^{va}} \delta_{f,a}^{va} * ADFD_{f,a} * FD_{f,a}^{-\rho_a^{va}}\right]^{-1/\rho_a^{va}}$	a	QVA_a	a
QVAFOC$_{f,a}$	$WF_f * WFDIST_{f,a} * \left(1 + TF_{f,a}\right) = PVA_a * QVA_a * AD_a^{va} * \left[\sum_{f \$ \delta_{f,a}^{va}} \delta_{f,a}^{va} * ADFD_{f,a} * FD_{f,a}^{-\rho_a^{va}}\right]^{-1} * \delta_{f,a}^{va} * ADFD_{f,a}^{-\rho_a^{va}} * \delta_{f,a}^{va} * FD_{f,a}^{\left(-\rho_a^{va} - 1\right)}$	$(f*a)$	$FD_{f,a}$	$(f*a)$
ADFAGEQ$_{f,a,r}$	$ADFAG_{ff,a} = \left(adfagb_{ff,a} + dabfag_{ff,a}\right) * \left(ADFAGfADJ_{ff} * ADFAGaADJ_a\right)$	$(fag*a)$		$(fag*a)$
FDPRODFN$_{ff,a}$	$FD_{ff,a} = AD_{ff,a}^{fag} * \left[\sum_{f \$ \delta_{ff,l,a}^{fd}} \delta_{ff,l,a}^{fd} * FD_{l,a}^{-\rho_{ff,a}^{fd}}\right]^{-1/\rho_{ff,a}^{fd}}$ $\forall FD0_{ff,a}$ and fag_{ff}	$(fag*a)$		$(fag*a)$
FDFOC$_{ff,l,a}$	$WF_l * WFDIST_{l,a} * \left(1 + TF_{l,a}\right) = WF_{ff} * WFDIST_{ff,a} * \left(1 + TF_{ff,a}\right) * FD_{ff,a} * \left[\sum_{l \$ \delta_{ff,l,a}^{fd}} \delta_{ff,l,a}^{va} * FD_{l,a}^{-\rho_{ff,a}^{fd}}\right]^{-1} * \delta_{ff,l,a}^{va} * FD_{l,a}^{\left(-\rho_{ff,a}^{fd} - 1\right)}$ $\forall \delta_{ff,l,a}^{va}$ and fag_{ff}	$(fag*l*a)$		$(fag*l*a)$

Name	Equation	Number of Equations	Variable	Number of Variables
$QINTDEQ_c$	$QINTD_c = \sum_a ioqtdqd_{c,a} * QINT_a$	c	$QINTD_c$	c
$COMOUT_c$	$QXC_c = adxc_c * \left[\sum_{a\$\delta_{a,c}^{xc}} \delta_{a,c}^{xc} * QXAC_{a,c}^{-\rho_c^{xc}} \right]^{-1/\rho_c^{xc}} \quad \forall cx_c \text{ and } cxac_c$	c	QXC_c	c
$COMOUT2_c$	$QXC_c = \sum_a QXAC_{a,c} \quad \forall cx_c \text{ and } cxacn_c$			
$COMOUTFOC_{a,c}$	$PXAC_{a,c} = PXC_c * QXC_c * \left[\sum_{a\$\delta_{a,c}^{xc}} \delta_{a,c}^{xc} * QXAC_{a,c}^{-\rho_c^{xc}} \right]^{-\left(\frac{1+\rho_c^{xc}}{\rho_c^{xc}}\right)} * \delta_{a,c}^{xc} * QXAC_{a,c}^{\left(-\rho_c^{xc}-1\right)} \quad \forall \delta_{a,c}^{xc} \text{ and } cxac_c$	$(a*c)$	$PXAC_{a,c}$	$(a*c)$
$COMOUT2FOC_{a,c}$	$PXAC_{a,c} = PXC \quad \forall cxacn_c$			
$ACTOUT_{a,c}$	$QXAC_{a,c} = IOQXACQX_{a,c} * QX_a \quad \forall IOQXACQX_{a,c}$ **and** $acetn_a$	$(a*c)$	$QXAC_{a,c}$	$(a*c)$
$ACTOUTFOC_{a,c}$	$QXAC_{a,c} = QX_a * \left(\frac{PXAC_{a,c}}{\left(PX_a * gamma_{a,c}^i * at_a^{i\,\rho_a^i}\right)} \right)^{\left(1/\left(\rho_a^i - 1\right)\right)} \quad \forall IOQXACQX_{a,c}$ **and** $acet_a$			

Name	Equation	Number of Equations	Variable	Number of Variables
	FACTOR BLOCK			
$YFEQ_f$	$YF_f = \left(\sum_a WF_f * WFDIST_{f,a} * FD_{f,a}\right) + \left(factwor_f * ER\right)$	*f*	YF_f	*f*
$YFDISPEQ_f$	$YFDISP_f = \left(YF_f * \left(1 - deprec_f\right)\right) * \left(1 - TYF_f\right)$	*f*	$YFDIST_f$	*f*
$YFINSEQ_f$	$YFINS_f = YFDISP_f - \left[\left(govvash_f * YFDISP_f\right) + \left(worvash_f * YFDISP_f\right)\right]$	*f*	$YFINS_f$	*f*
$FSISHEQ_{insw,f}$	$FSISH_{insw,f} = FSI_{insw,f} \Big/ \sum_{insw} FSI_{insw,f}$	*(insw*f)*	$FSISH_{insw,f}$	*(insw*f)*
$INSVASHEQ_{insw,f}$	$INSVA_{insw,f} = FSISH_{insw,f} * YFINS_f$	*(insw*f)*	$INSVASH_{insw,f}$	*(insw*f)*

Name	Equation	Number of Equations	Variable	Number of Variables
	HOUSEHOLD BLOCK			
YHEQ$_h$	$YH_h = \left(\sum_f INSVA_{h,f}\right) + \left(\sum_{hp} HOHO_{h,hp}\right) + HOENT_h + \left(hogovconst_h * HGADJ * CPI\right) + \left(howor_h * ER\right)$	*h*	YH_h	*h*
HOHOEQ$_{h,hp}$	$HOHO_{h,hp} = hohosh_{h,hp} * \left(YH_h * \left(1 - TYH_h\right)\right) * \left(1 - SHH_h\right)$	*h*hp*	$HOHO_{h,hp}$	*h*hp*
HEXPEQ$_h$	$HEXP_h = \left(\left(YH_h * \left(1 - TYH_h\right)\right) * \left(1 - SHH_h\right)\right) - \left(\sum_{hp} HOHO_{hp,h}\right)$	*h*	$HEXP_h$	*h*

Name	**Equation**	***Number of Equations***	**Variable**	***Number of Variables***
	HOUSEHOLD BLOCK			
QCDLES1EQc	$QCD_{cc,h} = \frac{\sum_{h}\left(\left(PQD_{cc}*(1+TV_{cc})*qcdconst_{cc,h}\right)+\sum_{h} beta_{cc,h} * \begin{pmatrix} HEXP_{h}-\sum_{c}\left(PQCD_{cag,h}*qcdconst_{cag,h}\right) \\ -\sum_{c}\left(PQD_{cc}*(1+TV_{cc})*qcdconst_{cc,h}\right)\end{pmatrix}\right)}{\left(PQD_{cc}*(1+TV_{cc})\right)}$ $\forall ccesn(cc)$	*cles*	QCD_{cles}	*cles*
QCDLES1EQc	$QCD_{cag,h}*PQCD_{cagc} = \left(qcdconst_{cc,h}*PQCD_{cc}\right) + beta_{cag,h} * \begin{pmatrix} HEXP_{h}-\sum_{cag}\left(PQCD_{cag,h}*qcdconst_{cag,h}\right) \\ -\sum_{ccesn}\left(PQD_{ccesn}*(1+TV_{ccesn})*qcdconst_{ccesn,h}\right)\end{pmatrix}$			
QCDCESEQc	$QCD2_{cag,cc,h} = QCD_{cag,h} * \left(\frac{\left(\left(PQD_{cc}*(1+TV_{cc})\right)*accd_{cag,h}{}^{-\rho^{cd}_{cag,h}}\right)}{\left(PQCD_{cag,h}*\delta^{cd}_{cag,cc,h}\right)}\right)^{\left(-1/\left(\rho^{cd}_{cag,h}+1\right)\right)}$ $\forall cces(cc)$	*cces*	QCD_{cces}	*cces*

Name	Equation	Number of Equations	Variable	Number of Variables
	ENTERPRISE BLOCK			
YEEQ	$YE_e = \left(\sum_f INSVA_{e,f} \right) + \left(entgovconst_e * EGADJ * CPI \right) + \left(entwor_e * ER \right)$	1	*YE*	1
QENTDEQ$_c$	$QED_{c,e} = qedconst_{c,e} * QEDADJ$	*c*	*QENTD*$_c$	*c*
VENTDEQ	$VED_e = \left(\sum_c QED_{c,e} * PQD_c \right)$	1	*VENTD*	1
HOENTEQ$_h$..	$HOENT_{h,e} = hoentsh_{h,e} * \left(\left(YE_e * \left(1 - TYE_e\right)\right) * \left(1 - SEN_e\right) - \sum_c \left(QED_{c,e} * PQD_c \right) \right)$	*h*	*HOENT*$_h$	*h*
GOVENTEQ..	$GOVENT_e = goventsh_e * \left(\left(YE_e * \left(1 - TYE_e\right)\right) * \left(1 - SEN_e\right) - \sum_c \left(QED_c * PQD_c \right) \right)$	1	*GOVENT*	1

Name	Equation	Number of Equations	Variable	Number of Variables
	TAX RATE BLOCK			
$TMDEF_c$	$TM_c = ((tmb_c + dabtm_c)*TMADJ) + (DTM*tm01_c)$	*cm*	*TM*	*cm*
$TEDEF_c$	$TE_c = ((teb_c + dabte_c)*TEADJ) + (DTE*te01_c)$	*ce*	*TE*	*ce*
$TSDEF_c$	$TS_c = ((tsb_c + dabts_c)*TSADJ) + (DTS*ts01_c)$	*c*	*TS*	*c*
$TQSDEF_c$	$TQS_c = ((tqsb_c + dabtqs_c)*TQSADJ) + (DTQS*tqs01_c)$	*c*	*TQS*	*c*
$TVDEF_c$	$TV_c = ((tvb_c + dabtv_c)*TVADJ) + (DTV*tv01_c)$	*c*	*TV*	*c*
$TEXDEF_c$	$TEX_c = ((texb_c + dabtex_c)*TEXADJ) + (DTEX*tex01_c)$	*c*	*TEX*	*c*
$TXDEF_a$	$TX_a = ((txb_a + dabtx_a)*TXADJ) + (DTX*tx01_a)$	*a*	*TX*	*a*
$TFDEF_{f,a}$	$TF_{f,a} = ((tfb_{f,a} + dabtf_{f,a})*TFADJ) + (DTF*tf01_{f,a})$	*f*a*	*TF*	*f*a*
$TYFDEF_f$	$TYF_f = ((tyfb_f + dabtyf_f)*TYFADJ) + (DTYF*tyf01_f)$	*f*	*TYF*	*f*
$THYDEF_h$	$TYH_h = ((tyhb_h + dabtyh_h)*TYHADJ) + (DTYH*tyh01_h)$	*h*	*TYH*	*h*
$TYEDEF_e$	$TYE_e = ((tyeb_e + dabtye_e)*TYEADJ) + (DTYE*tye01_e)$	*e*	*TYE*	*e*

Name	Equation	Number of Equations	Variable	Number of Variables
	TAX REVENUE BLOCK			
MTAXEQ	$MTAX = \sum_{c}\left(TM_c * PWM_c * ER * QM_c\right)$	1	*MTAX*	1
ETAXEQ	$ETAX = \sum_{c}\left(TE_c * PWE_c * ER * QE_c\right)$	1	*ETAX*	1
STAXEQ	$STAX = \sum_{c}\left(TS_c * PQS_c * QQ_c\right)$	1	*STAX*	1
QSTAXEQ	$QSTAX = \sum_{c}\left(TQS_c * QQ_c\right)$	1	*QSTAX*	1
VTAXEQ	$VTAX = \sum_{h}\sum_{c}\left(TV_c * PQD_c * QCD_{c,h\$ccesn_c}\right) + \sum_{cag}\sum_{h}\sum_{c}\left(TV_c * PQD_c * QCD2_{cag,c,h\$cces_c}\right)$	1	*VTAX*	1
EXTAXEQ	$EXTAX = \sum_{c}\left(TEX_c * PQS_c * QQ_c\right)$	1	*EXTAX*	1
ITAXEQ	$ITAX = \sum_{a}\left(TX_a * PX_a * QX_a\right)$	1	*ITAX*	1
FTAXEQ	$FTAX = \sum_{f,a}\left(TF_{f,a} * WF_f * WFDIST_{f,a} * FD_{f,a}\right)$	1	*FTAX*	1
FYTAXEQ	$FYTAX = \sum_{f}\left(TYF_f * \left(YF_f * \left(1 - deprec_f\right)\right)\right)$	1	*FTAX*	1
DTAXEQ	$DTAX = \sum_{h}\left(TYH_h * YH_h\right) + \sum_{e}\left(TYE_e * YE\right)$	1	*DTAX*	1

Name	Equation	Number of Equations	Variable	Number of Variables
	GOVERNMENT BLOCK			
YGEQ	$YG = MTAX + ETAX + STAX + EXTAX + VTAX + FTAX + ITAX + FYTAX + DTAX + \left(\sum_f INSVA_{g,f} \right) + GOVENT + \left(govwor * ER \right)$	1	*YG*	1
QGDEQc	$QGD_c = qgdconst_c * QGDADJ$	*c*	QGD_c	*c*
VGDEQ	$VGD = \left(\sum_c QGD_c * PQD_c \right)$	1	*VQGD*	1
EGEQ	$EG = \left(\sum_c QGD_c * PQD_c \right) + \left(\sum_h hogovconst_h * HGADJ * CPI \right) + \left(\sum_e entgovconst_e * EGADJ * CPI \right)$	1	*EG*	1

Name	Equation	Number of Equations	Variable	Number of Variables
	INVESTMENT BLOCK			
$SHHDEF_h$	$SHH_h = ((shhb_h + dabshh_h) * SHADJ * SADJ) + (DSHH * DS * shh01_h)$	h	SHH	H
$SENDEF_e$	$SEN_e = ((sen_e + dabsen_e) * SEADJ * SADJ) + (DSEN * DS * sen01_e)$	e	SEN	e
TOTSAVEQ	$TOTSAV = \sum_h ((YH_h * (1 - TYH_h)) * SHH_h)$ $+ \sum_e ((YE * (1 - TYE_e)) * SEN_e)$ $+ \sum_f (YF_f * deprec_f) + KAPGOV + (CAPWOR * ER)$	1	$TOTSAV$	1
$QINVDEQ_c$	$QINVD_{c,i} = (QINV_i * ioqinvd_{c,i})$ $QINV_i = qinvb_i * IADJ$ $INVEST * INVSH\_I_i = \sum_c PQD_c * QINVD_{c,i}$	c	$QINVD_c$	c
INVEST	$INVEST = \sum_{c,i} PQD_c * QINVD_{c,i}$	1	$INVEST$	1
	FOREIGN INSTITUTIONS BLOCK			
$YFWOREQ_f$	$YFWOR_f = \sum_w INSVA_{w,f}$	f	$YFWOR_f$	f

Name	Equation	Number of Equations	Variable	Number of Variables
	FACTOR MARKET CLEARING BLOCK			
FMEQUIL$_f$	$\sum_{insw} FSI_{insw,f} = \sum_{a} FD_{f,a}$	f	FS_f	f
	$FS_f = \sum_{ins} FSI_{ins,f}$			
	$FSIL_{insw,f} = \sum_{alei\$map\_hh\_alei_{ins,alei}} FD_{f,alei}$			
	INSTITUTION MIGRATION BLOCK			
	$YMIGR_{ins,insp} = \frac{\left[\sum_{f} INSVA_{insp,f}\right]}{\left[\sum_{f} INSVA_{ins,f}\right]} \quad \forall map\_insw\_inswp_{ins,insp}$			
	$FSIM\_I_{f,ins,insp} = fsia_{ins,f} - \sum_{insp\$(notsameas_{ins,insp})} FSIM\_I_{f,ins,insp}$ $\forall inswmig_{ins}$ **AND** $fsia_{ins,f}$			
	$FSI\_I_{ins,f} = \sum_{insp\$inswmig_{insp}} FSIM\_I_{f,insp,ins} \quad \not\forall inswmigp_{ins}$ and $FSI\_IA_{ins,f}$			
	$FSI\_I_{insw,f} = \sum_{inswp\$inswmig_{insw}} FSIM\_I_{f,insw,pinsw}$ $\forall inswmig_{insw}$ **AND** $FSI\_I0_{insw,f}$			

Name	Equation	Number of Equations	Variable	Number of Variables
	FACTOR MOBILITY BLOCK			
	$$WMIGR_{f,fp,ins} = \frac{\left[\left(\sum_{a} WF_{fp} * WFDIST_{fp,a} * FD_{fp,a}\right) \Big/ \sum_{a} FD_{fp,a}\right]}{\left[\left(\sum_{a} WF_{f} * WFDIST_{f,a} * FD_{f,a}\right) \Big/ \sum_{a} FD_{f,a}\right]}$$ $$\forall \left(map\_fmig\_fmigp_{f,fp} \textbf{ AND } FSI\_I0_{ins,f} \textbf{ AND } FSI\_I0_{ins,fp} \right)$$			
	$$WMIGR_{f,fp,ins} = WMIGRA_{f,fp,ins}$$ $$\not\forall \left(map\_fmig\_fmigp_{f,fp} \textbf{ AND } FSI\_I0_{ins,f} \textbf{ AND } FSI\_I0_{ins,fp} \right)$$			
	$$FSIM\_F_{f,fp,ins} = FSI\_I_{ins,f} * \left(\frac{WMIGR_{f,fp,ins}}{WMIGRA_{f,fp,ins}} \right)^{etaff_{f,fp,ins}} - FSI\_I_{ins,f}$$ $$\forall \left(map\_fmig\_fmigp_{f,fp} \textbf{ AND } FSI\_I0_{ins,f} \textbf{ AND } FSI\_I_{ins,fp} \right)$$			
	$$FSIM\_F_{f,f,ins} = FSI\_I_{ins,f} - \sum_{fp\$\left(notsameas_{fp,f}\right)} FSIM\_F_{f,fp,ins}$$ $$\forall fmig_{f} \textbf{ AND } FSI\_I0_{ins,f}$$			
	$$FSI_{ins,f} = \sum_{fp\$ fmig_{fp}} FSIM\_F_{fp,f,ins} \qquad \forall fmig_{f} \textbf{ AND } FSI0_{ins,f}$$			

Name	Equation	Number of Equations	Variable	Number of Variables
	COMMODITY MARKET CLEARING BLOCK			
$PRODEQUIL_{a,c}$	$QXAC_{a,c} = IOQXACQX_{a,c} + QX_a$	(a,c)	$QXAC$	(a,c)
$QEQUIL_c$	$QQ_c = QTTD_c + QINTD_c + \sum_h QCD_{c,h\$ccesn_c} + \sum_{cag} \sum_h QCD2_{cag,c,h\$cces_c} + \sum_e QED_{c,e} + QGD_c + \sum_i QINVD_{i,c}$	c		
	MACROECONOMIC CLOSURE BLOCK BLOCK			
$CAPGOVEQ$	$KAPGOV = YG - EG$	1	$CAPGOV$	1
$CAEQUIL$	$CAPWOR = \left(\sum_c PWM_c * QM_c\right) + \left(\sum_f \frac{YFWOR_f}{ER}\right) - \left(\sum_c PWE_c * QE_c\right) - \left(\sum_f factwor_f\right) - \left(\sum_h howor_h\right) - entwor - govwor$	1	$CAPWOR$	1

Name	Equation	Number of Equations	Variable	Number of Variables
	ABSORPTION CLOSURE BLOCK			
VFDOMDEQ	$VFDOMD = \left(\sum_{c,h} PQD_c * (1+TV_c) * QCD_{c,h\$ccesn_c}\right) + \left(\sum_{cag,h} PQD_c * (1+TV_c) * QCD2_{cag,c,h\$cces_c}\right) + \left(\sum_{e,c} PQD_c * QED_{e,c}\right) + \left(\sum_{c} PQD_c * QGD_c\right) + \left(\sum_{i,c} PQD_c * QINVD_{i,c}\right)$	1	*VFDOMD*	1
VEDSHEQ	$VEDSH_e = VED_e \big/ VFDOMD$	1	*VENTDSH*	1
VGDSHEQ	$VGDSH = VGD \big/ VFDOMD$	1	*VGDSH*	1
INVESTSHEQ	$INVESTSH = INVEST \big/ VFDOMD$	1	*INVESTSH*	1
	GDP BLOCK			
GDPEQ	$GDP = \left(\sum_{c,h} PQD_c * (1+TV_c) * QCD_{c,h\$ccesn_c}\right) + \left(\sum_{cag,h} PQD_c * (1+TV_c) * QCD2_{cag,c,h\$cces_c}\right) + \left(\sum_{e,c} PQD_c * QED_{e,c}\right) + \left(\sum_{c} PQD_c * QGD_c\right) + \left(\sum_{i,c} PQD_c * QINVD_{i,c}\right) + \left(\sum_{c} PWE_c * QE_c * ER\right) - \left(\sum_{c} PWM_c * QM_c * ER\right)$	1	*GDP*	1
	SLACK BLOCK			

WALRASEQ	$TOTSAV = INVEST + WALRAS$	1	*WALRAS*	1

Name	Equation	Number of Equations	Variable	Number of Variables
	MODEL CLOSURE			
			$\overline{ER}$ or $\overline{CAPWOR}$	1
			$\overline{PWM_c}$ and $\overline{PWE_c}$ or $\overline{PWE_{cedn}}$	2*c*
			$\overline{SADJ}, \overline{SHADJ}, \overline{SEADJ}$ or $\overline{IADJ}$ or $\overline{INVEST}$ or $\overline{INVESTSH}$	1
			$\overline{QEDADJ}$ or $\overline{VED}$ or $\overline{VEDSH}$	1
			At least **one** of $\overline{TMADJ}$, $\overline{TEADJ}$, $\overline{TSADJ}$, $\overline{TEXADJ}$, $\overline{TFADJ}$, $\overline{TXADJ}$, $\overline{TFADJ}$, $\overline{TYHADJ}$, $\overline{TYEADJ}$ $\overline{DTM}, \overline{DTE}, \overline{DTS}, \overline{DTEX}, \overline{DTF}, \overline{DTX}, \overline{DTYF}, \overline{DTYH}, \overline{DTYE}$, and $\overline{CAPGOV}$	7
			at least **two** of $\overline{QGDADJ}$, $\overline{HGADJ}$, $\overline{EGADJ}$, $\overline{VGD}$ and $\overline{VGDSH}$	3
			$\overline{FS_f}$ and $\overline{WFDIST_{f,a}}$	(*f**(*a*+1))
			$\overline{CPI}$ or $\overline{PPI}$	1